International Graduate School on Control

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International Summer School in Mathematical Control

M24 – SICHUAN
15/07/2019-27/07/2019

PDE Control: Theory, Numerics and Applications

Summary of the course

Partial Differential Equations (PDE) are the models to describe and formulate most of the challenging issues in Science and Technology. This is true not only in the classical contexts of structural or aeronautical engineering, for instance, but also in Mathematical Biology, Finances and Social Sciences. The first challenge is to derive an appropriate model. Then, issues related to the analysis of their solutions and numerical approximation arise. But, in most applications, control theoretical tools need also to be implemented to answer to the need of design, observation or actuation. In this series of lectures we shall introduce the main analytical tools on the control of Partial Differential Equations (PDE) and their numerical approximation and simulation. We shall also point towards some potential future perspectives of research.

We shall first present, from an historical perspective, some of the most relevant applications to Sciences, Engineering and Technology that have determined the development of the field. Then, after a short introduction to the finite-dimensional theory, we shall describe the basic theory for the wave and heat equations, the most elementary PDE models, to later address some important multi-physics models. Then we shall address the problem of the numerical approximation presenting some methods allowing to reduce the PDE control problem to a finite-dimensional one in an accurate manner. We shall also present some interesting concepts and results on switching, sparse, averaged and bang-bang control, and the turnpike property ensuring that, most often, in long-time horizons, optimal controls and trajectories are close to the steady-state ones. To conclude, we shall present a list of open problems and directions of possible future research.

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Outline of the Course

1. Historical introduction
2. Introduction to finite-dimensional control
3. Wave propagation
4. Heat diffusion
5. Some relevant models of multi-physics nature
6. Numerical approximation of control problems
7. Averaged control on parameter depending systems
8. Switching, sparse and bang-bang control
9. The turnpike property
10. Perspectives and open problems

References

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M25 – SICHUAN
08/07/2019-27/07/2019

Optimal Stochastic Control

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Summary of the course:

Optimal stochastic control is concerned with optimization problems for controlled stochastic differential equations with certain type cost/payoff functionals. There are quite a few interesting applications in various applied areas, including mathematical finance, communication networks, engineering problems involving random disturbances, bio-mathematics, to mention a few.

In this course, we will briefly present some basic results in optimal stochastic control theory. Beside a general formulation of optimal control problem as well as a quick review of stochastic analysis, the following will be presented: (i) Pontryagin type maximum principle for optimal controls and a theory of forward-backward stochastic differential equations; (ii) Bellman dynamic programming method and viscosity solution of Hamilton-Jacobi-Bellman equations; (iii) Linear-quadratic optimal control, open-loop and closed-loop solvability, and Riccati equations. Some new results obtained in the recent years will be included in the course.

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M26 – SICHUAN
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Stochastic PDE Control

Overview

Control of stochastic PDEs is developed to deal with the existence of uncertainty either in observations or in the noise that drives the evolution of the system governed by SPDEs. The aim of this theory is to design the control variables that drive the state to behave a desired performance, such as to arrive a given destination, or to minimize the cost, or to be insensitive with respect to the uncertainty, despite the presence of this noise. Stochastic control covers a broad area of disciplines and problems. It is also a field in full development, and some important aspects remain to be cleaned up. Over the past twenty-five years, stochastic control theory has been developing rapidly. In this course, we will briefly present some basic results and some new results obtained in the recent in control theory of SPDEs.

Outline

1. A quick review of stochastic analysis.
2. A general formulation of controllability and optimal control problems for SPDEs.
3. Exact/null controllability of some SPDEs.
4. Linear-quadratic optimal control, open-loop and closed-loop solvability, and Riccati equations.
5. Pontryagin type maximum principle for optimal controls and a theory of transposition solution to backward stochastic evolution equations.

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M27 – SICHUAN
08/07/2019-27/07/2019

Measurement and control of open quantum systems

Overview

Quantum control is an emerging research subject with an increasing role in technologies related to high precision metrology, quantum information, communication and computation. Its development requires to reconsider how measurement, control, and interactions fundamentally affect a system --- in particular, the intrinsic invasive character of measurements. This course presents some modern tools for controlling quantum systems, i.e. steering a system to a quantum state and stabilizing it against decoherence (dissipation of quantum information through the coupling of the system to its uncontrolled environment). These tools will be illustrated by recent feedback experiments in cavity and circuit quantum electrodynamics (QED). The context throughout is that of systems of ordinary and stochastic differential equations and the level will be that of a graduate course intended for a general control audience without any prerequisites in quantum mechanics.

Outline

1. Introduction to quantum mechanics: Schrödinger equation, two-level system (qubit, Pauli operators, Bloch sphere representation), harmonic oscillator (annihilation/creation operators, coherent state and displacement), composite systems (tensor product, spin-spring and Jaynes Cumming Hamiltonians), measurement back-action and wave function collapse.
2. Open-loop control of closed quantum systems: Lie algebra controllability results, resonant control, second order averaging and rotating waves approximation, adiabatic control and quantum annealing, optimal control and the GRAPE algorithm.
3. Discrete time open quantum systems: counting photons with atoms (cavity QED, LKB photon box), positive operator valued measures, Markov chain and quantum filtering, density operator formulation to include measurement imperfections and decoherence, Kraus maps and quantum channels, Quantum Non Demolition (QND) measurements, static output feedback (Markovian feedback) and quantum error correction, stabilization via measurement-based feedback, stabilization via autonomous feedback (reservoir and decoherence engineering).
4. Continuous time open quantum systems: weak measurement, QND measurements, quantum stochastic master equations, quantum filtering, Lindblad differential equations, decoherence of qubits and harmonic oscillators, Markovian feedback, measurement-based feedback and autonomous feedback in recent circuit QED experiments.

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