Mixing length computations for tracer flow in heterogeneous porous media by multiscale and lagrangian procedures

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Summary

Introduction

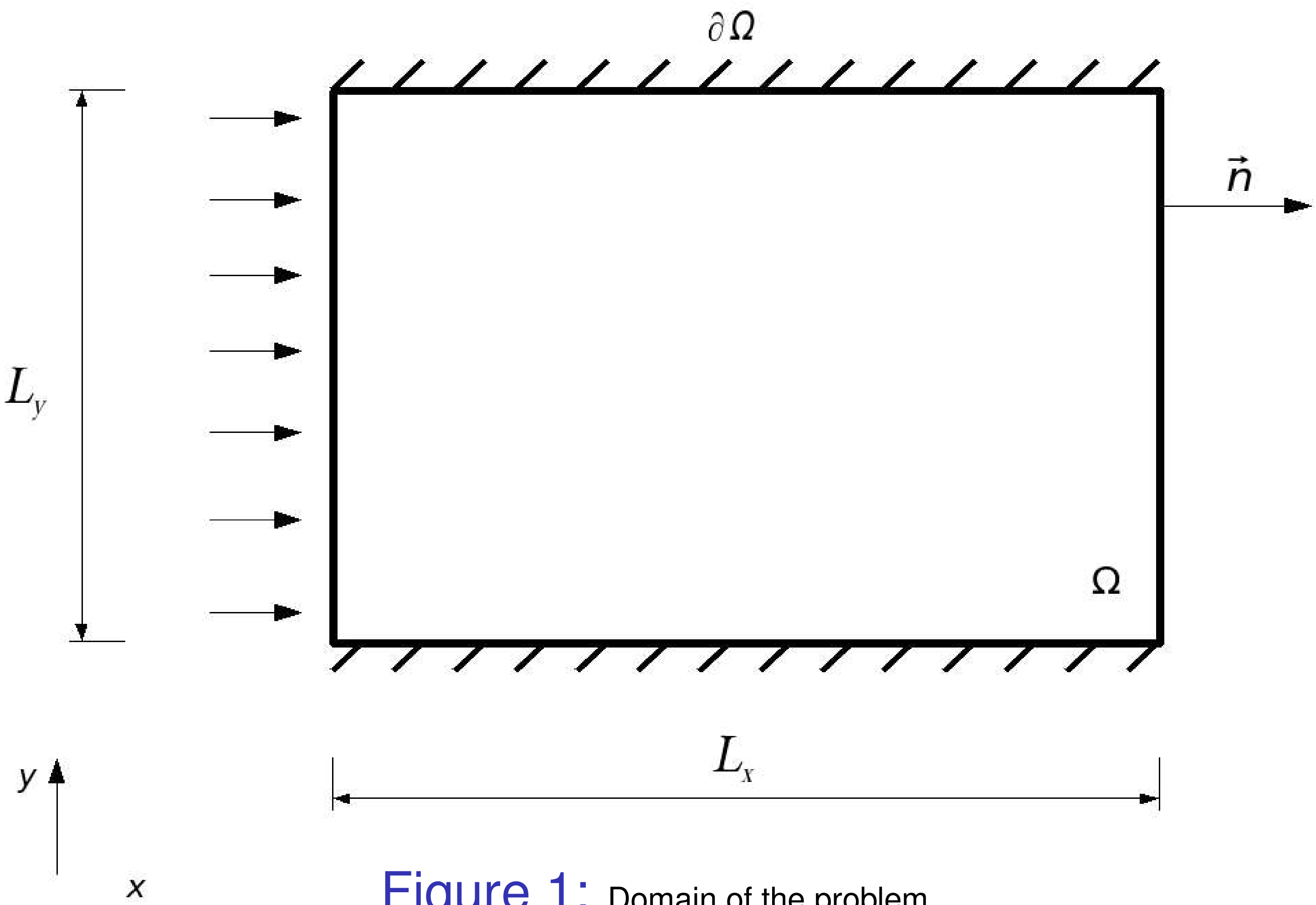
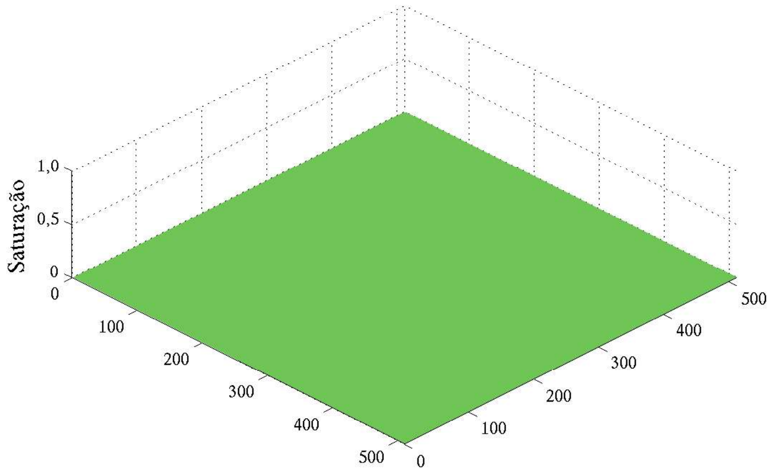
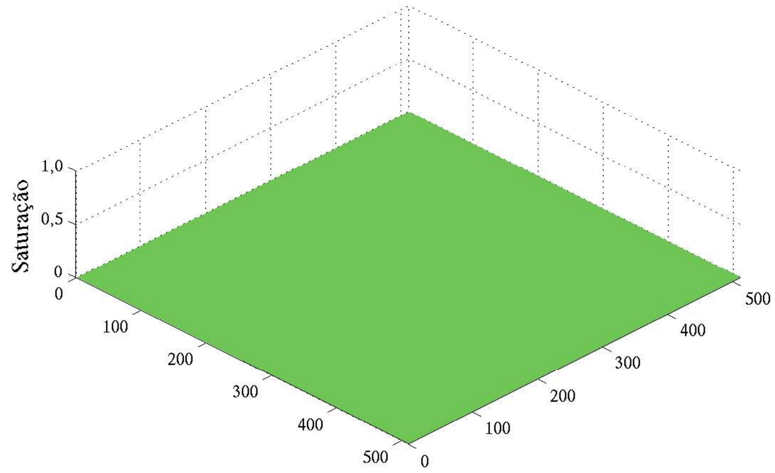


Figure 1: Domain of the problem.

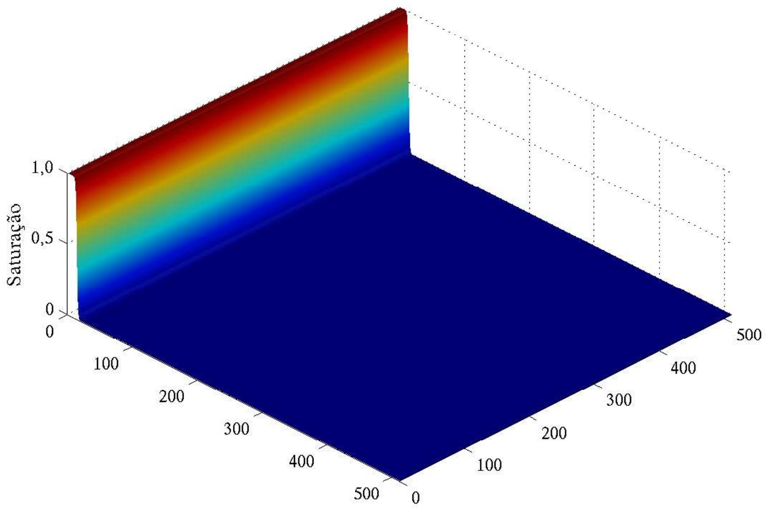
Homogeneous Heterogeneous

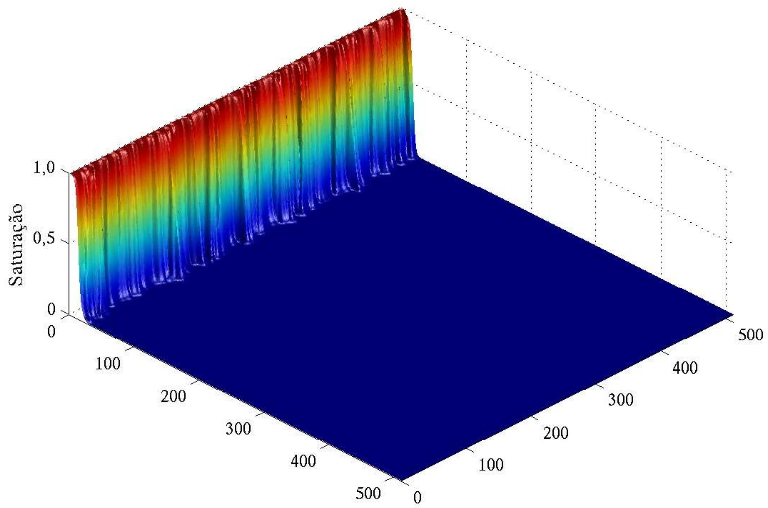




Homogeneous

Heterogeneous





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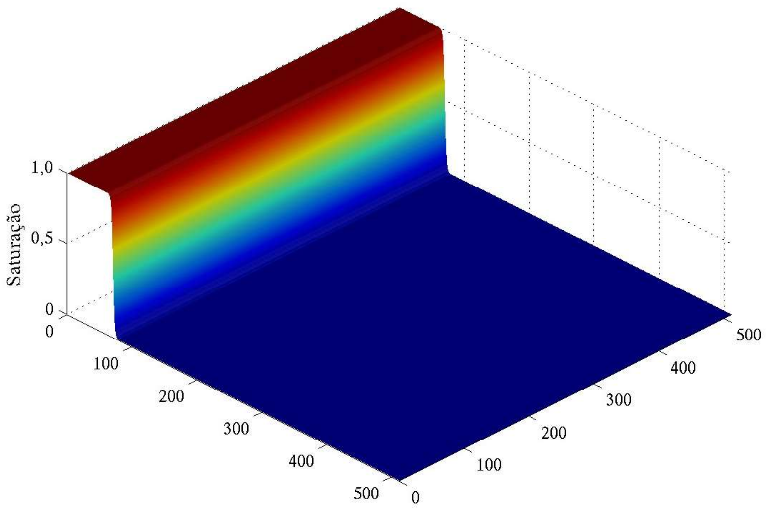

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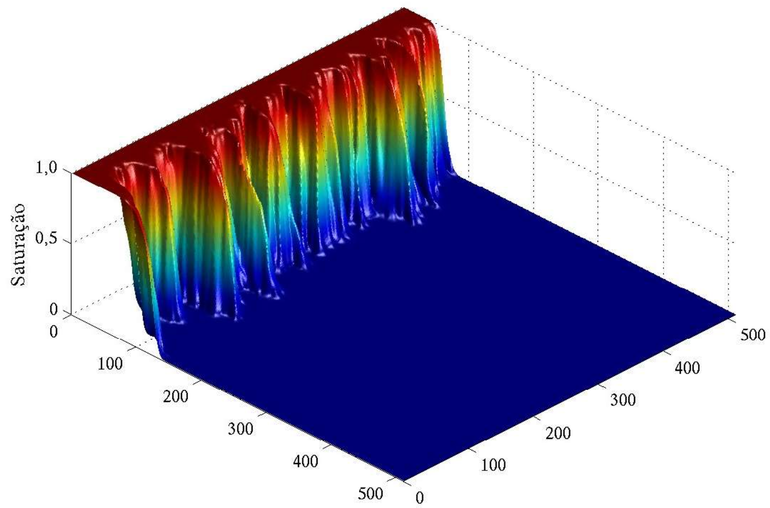

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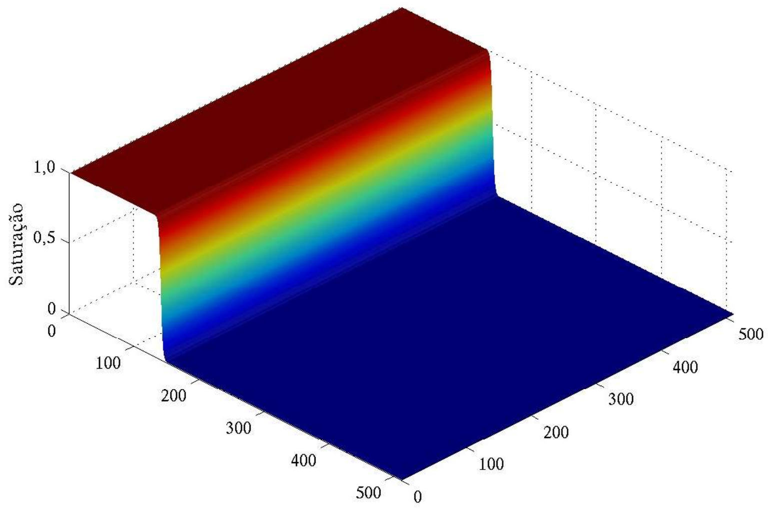

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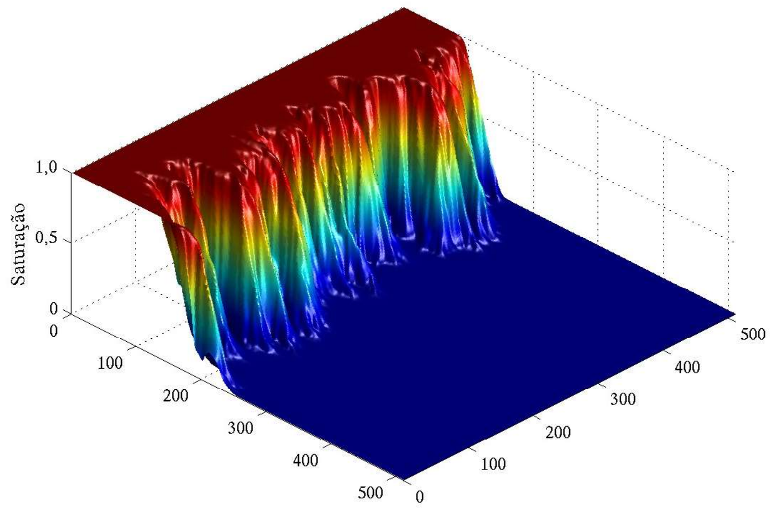

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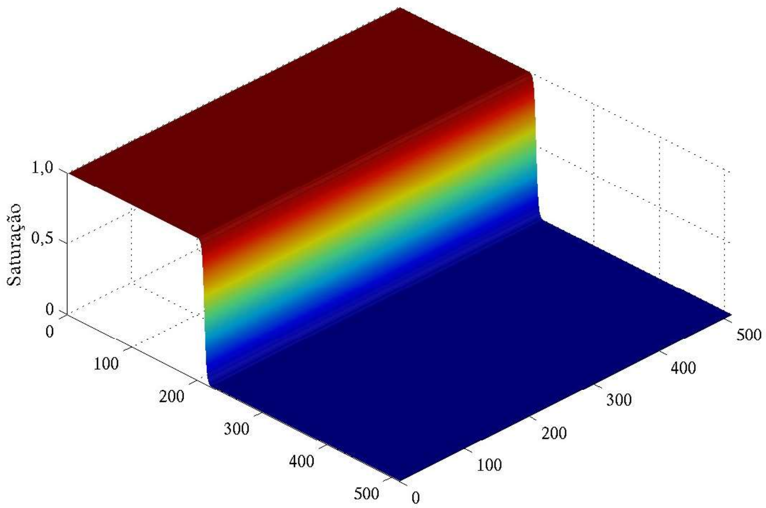

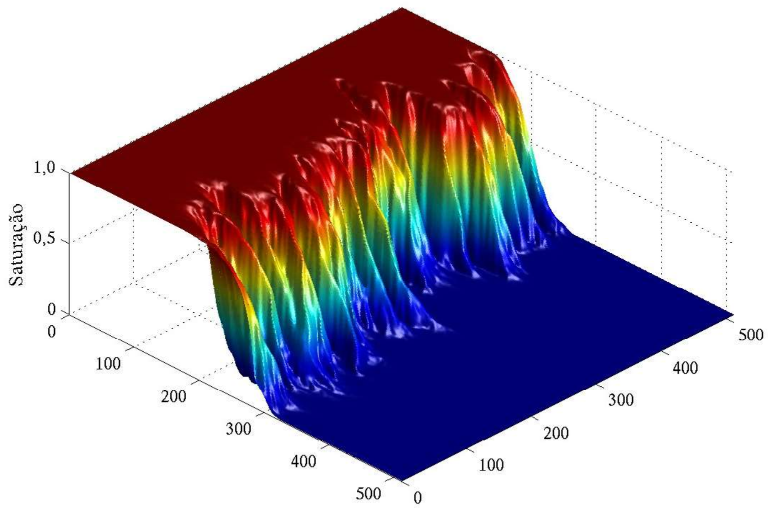

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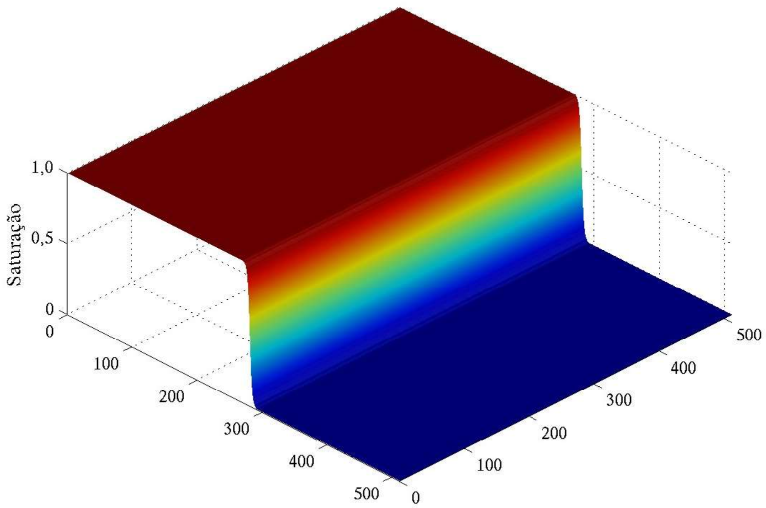

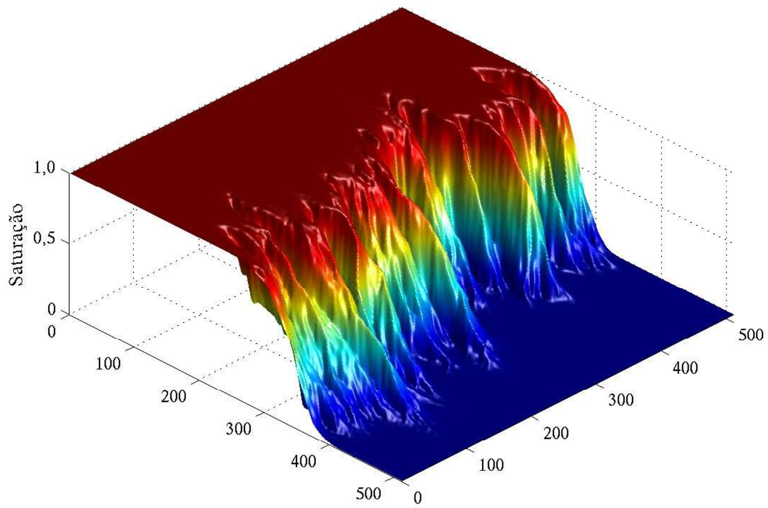

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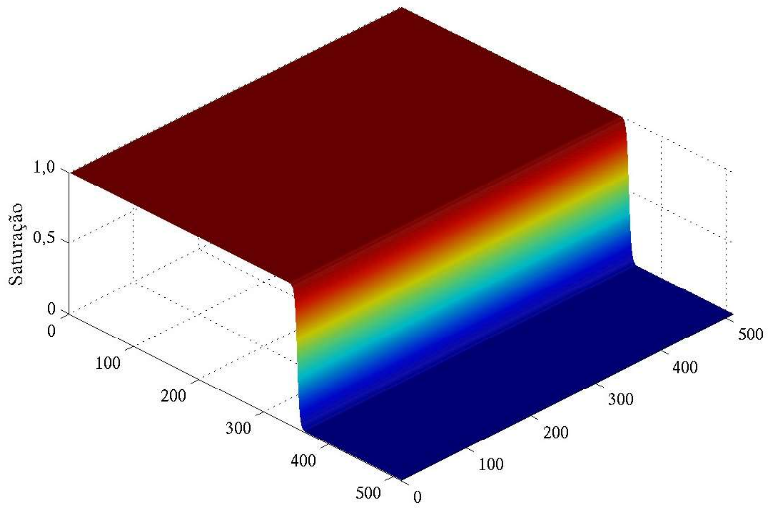

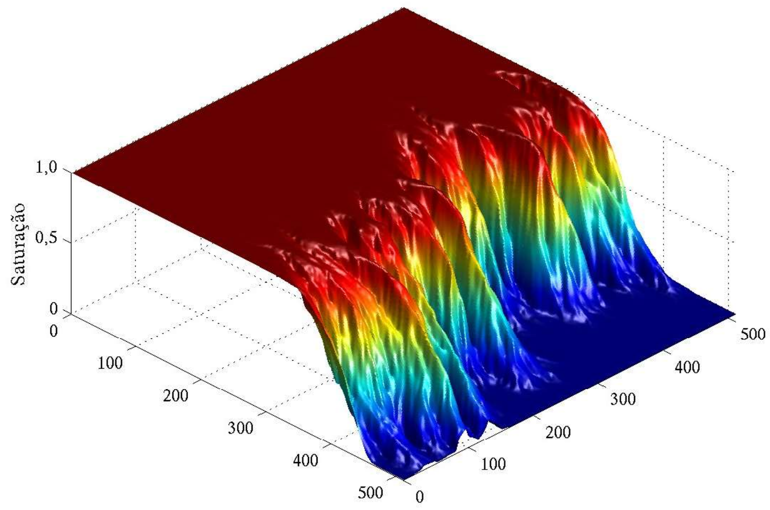

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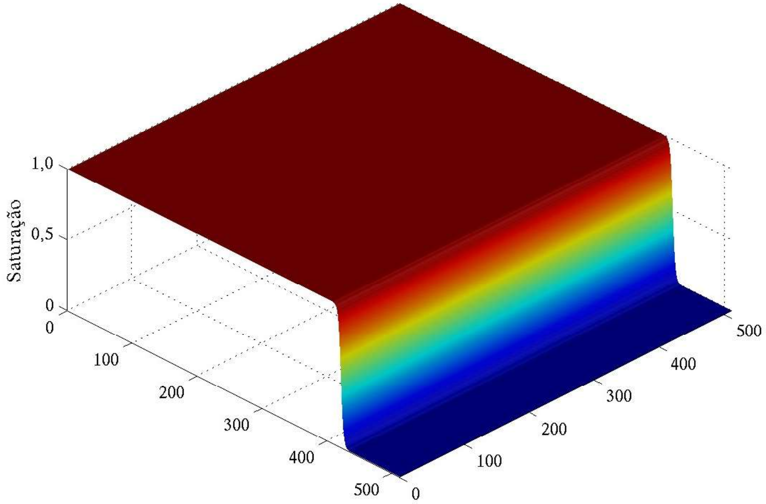

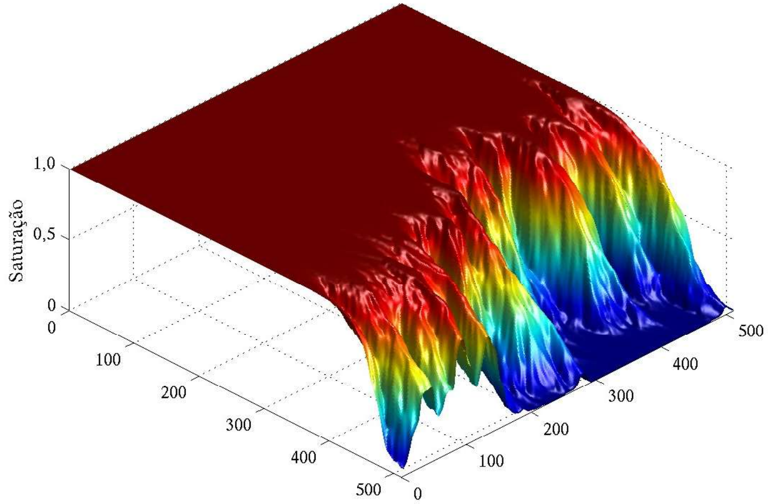

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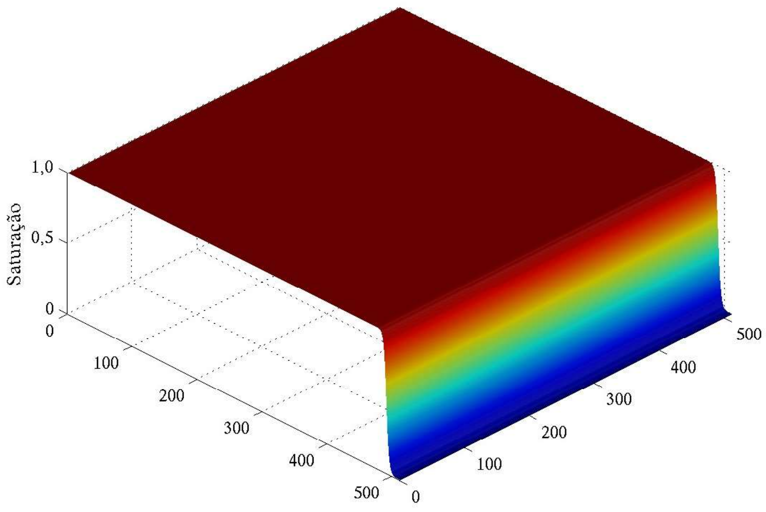

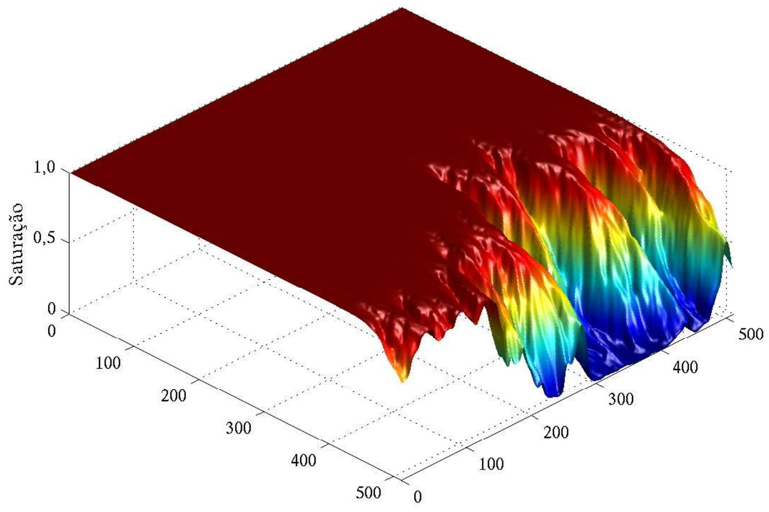

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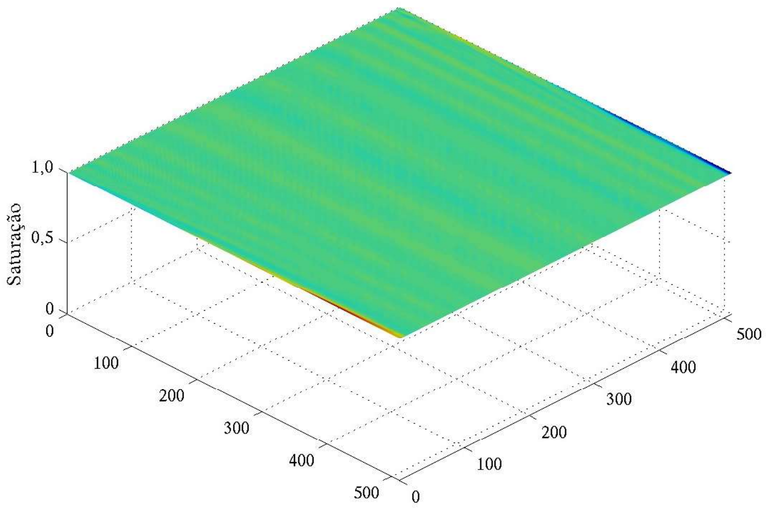

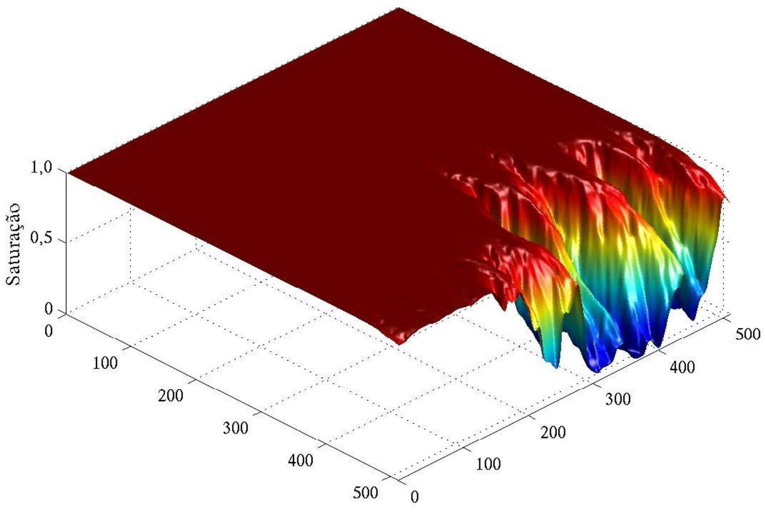

Governing equations

- The incompressible passive tracer flow

Flow equation

$$\mathbf{u} = \frac{-k(\mathbf{x})}{\mu} \nabla p, \qquad \nabla \cdot \mathbf{u} = 0, \tag{1}$$

- where
  - p is the pressure,
  - μ is the viscosity,
  - $\mathbf{u}$ is the velocity, and
  - $k(\mathbf{x})$ is the permeability field.

Perm.

Governing equations

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Perm.

Permeability field

- Furtado e Pereira (2003):

Stochastic permeability field (probabilistic model)

$$k(\mathbf{x}) = k_0 e^{\rho \xi(\mathbf{x})} \tag{2}$$

- mean: $\langle k(\mathbf{x}) \rangle = k_0$
- standard deviation: $\sigma = \rho$
- gaussian field: $\xi(\mathbf{x}) = ln\ k(\mathbf{x})$

Governing equations

Transport equation

$$\phi \frac{\partial c}{\partial t} + \mathbf{u} \cdot \nabla c = 0, \tag{3}$$

- where
  - c is the passive tracer concentration.

Initial and boundary conditions

- Domain $\Omega = [0, L_x] \times [0, L_y] \subset \mathbb{R}^2$,
- Time interval $I = [0, T]$

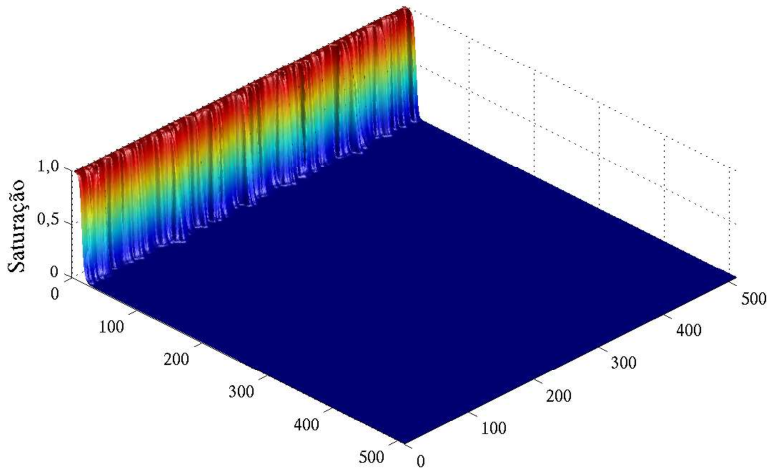
Boundary condition

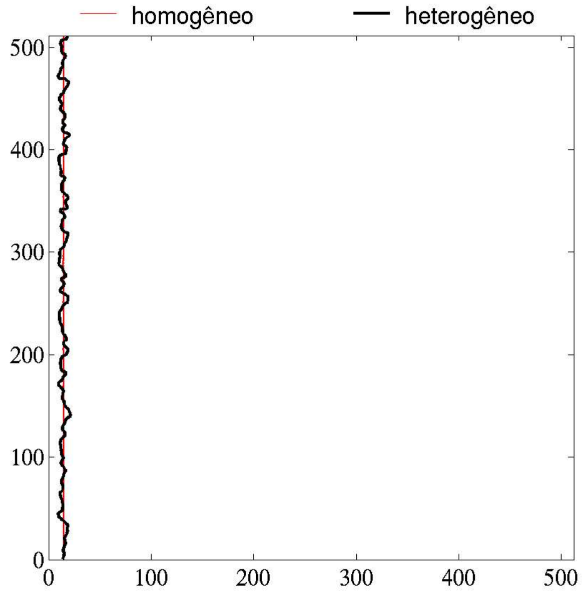
$$\begin{array}{llll} \mathbf{u} \cdot \mathbf{n} = -q, & on & x = 0, & y \in [0, L_y] \\ p = 0, & on & x = L_x, & y \in [0, L_y] \\ \mathbf{u} \cdot \mathbf{n} = 0, & on & y = 0, L_y, & x \in [0, L_x] \end{array} \quad (4)$$

Initial condition

$$c(\mathbf{x}, 0) = c_0(\mathbf{x}). \quad (5)$$

Macroscopic mixing length





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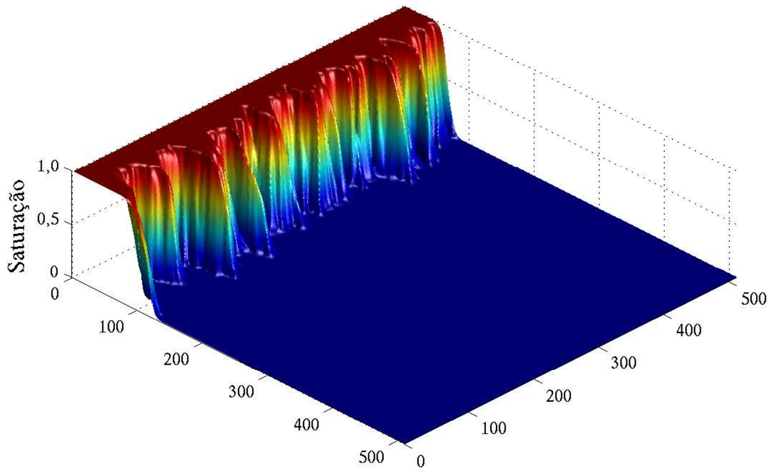

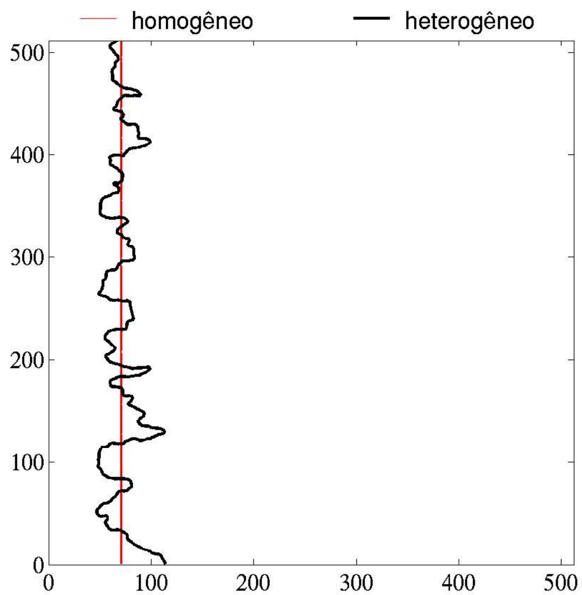

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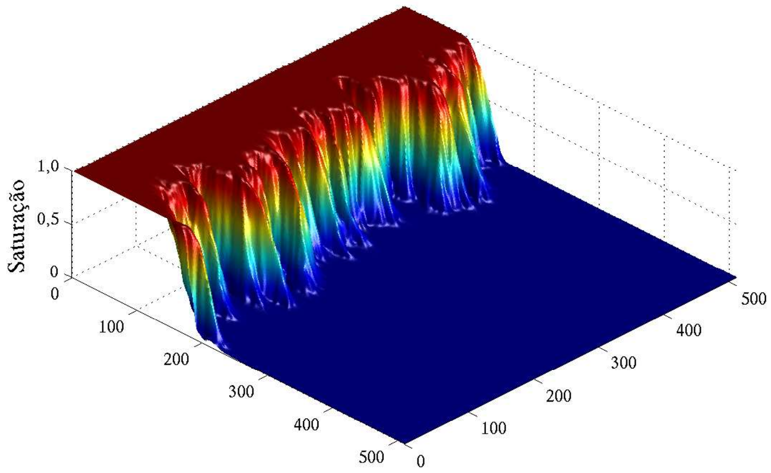

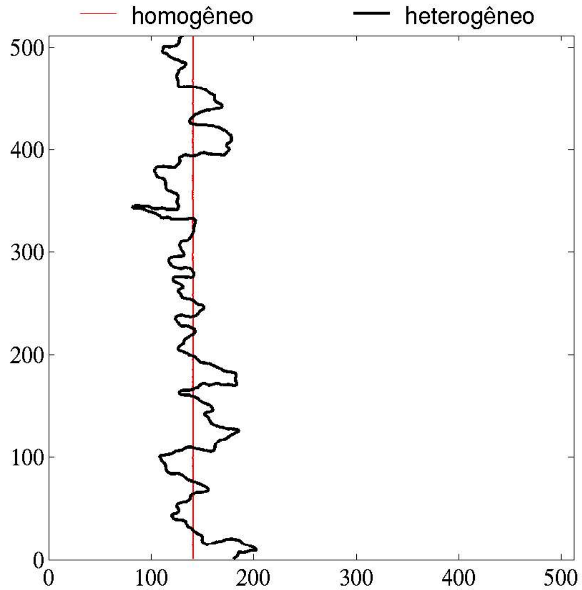

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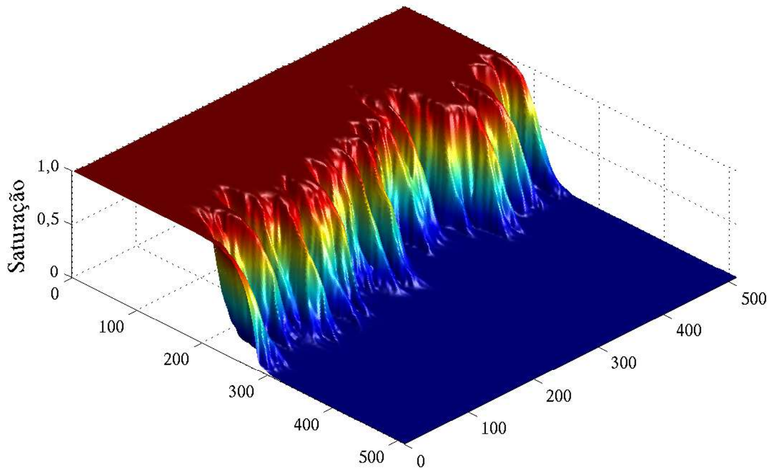

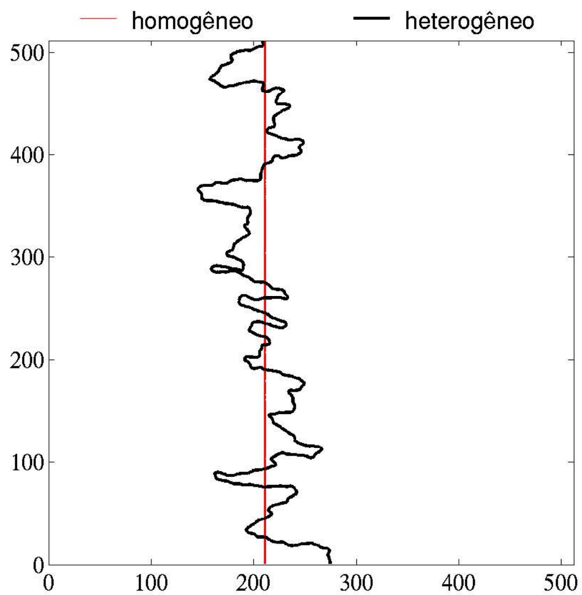

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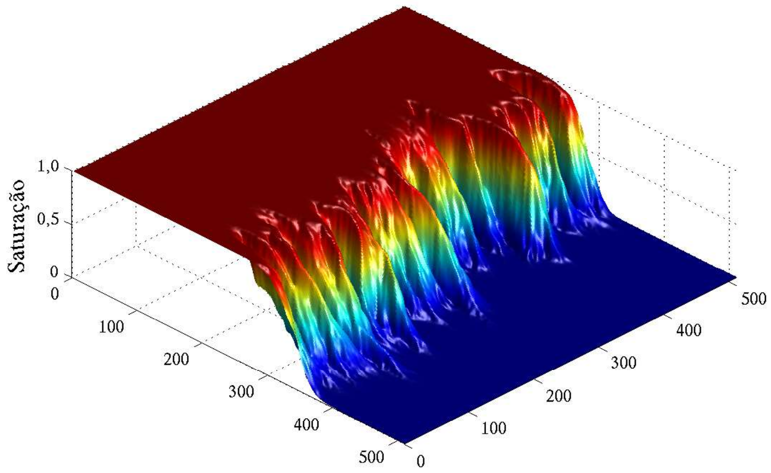

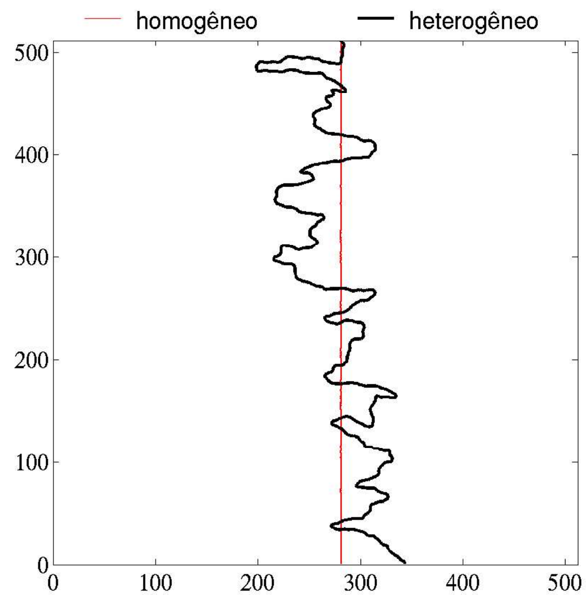

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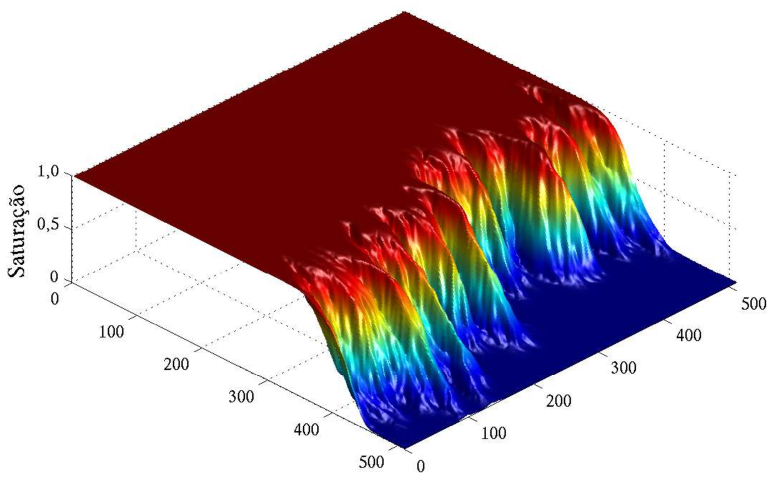

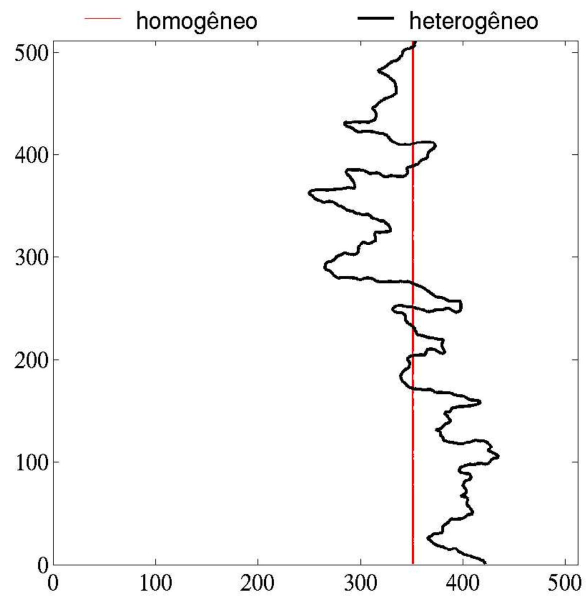

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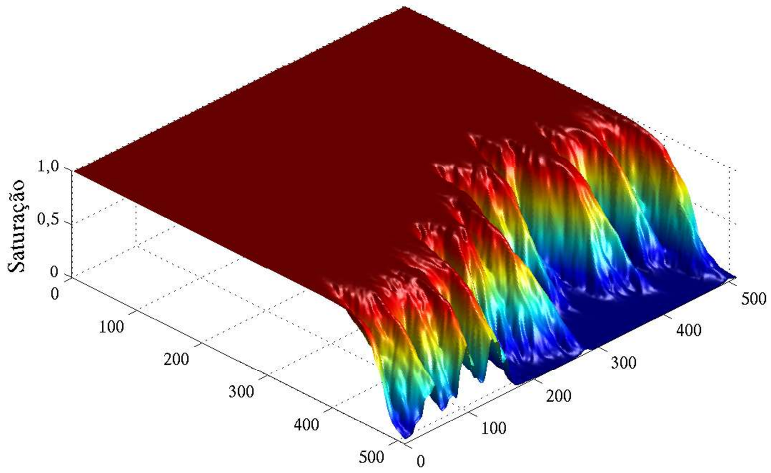

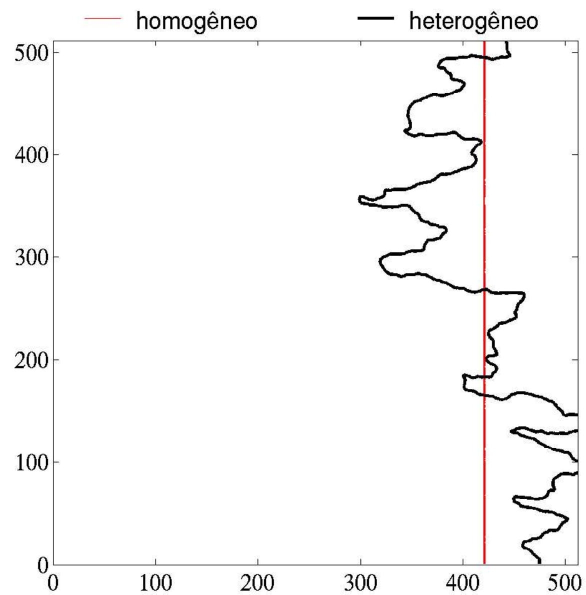

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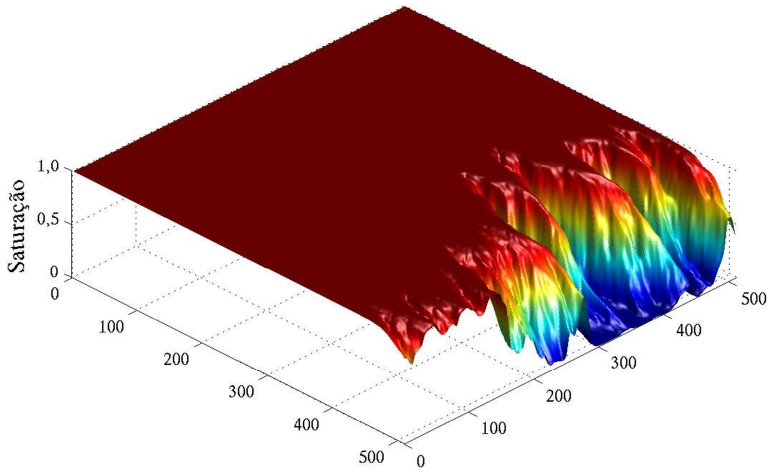

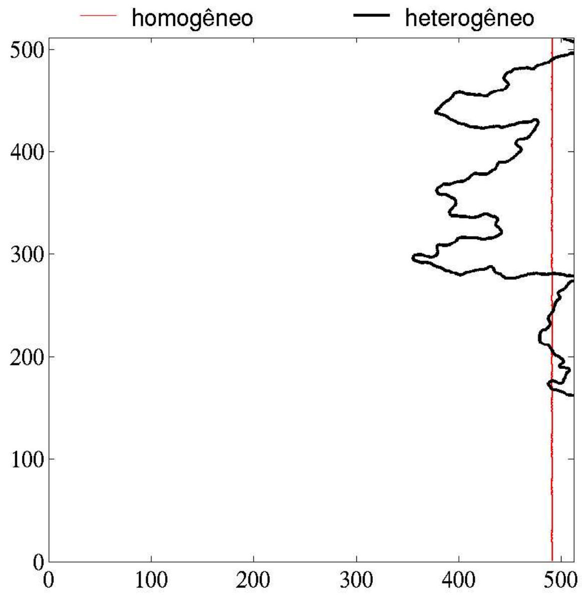

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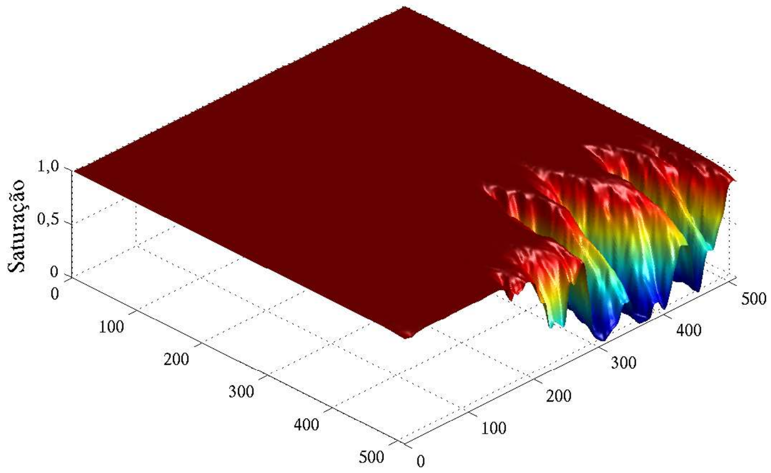

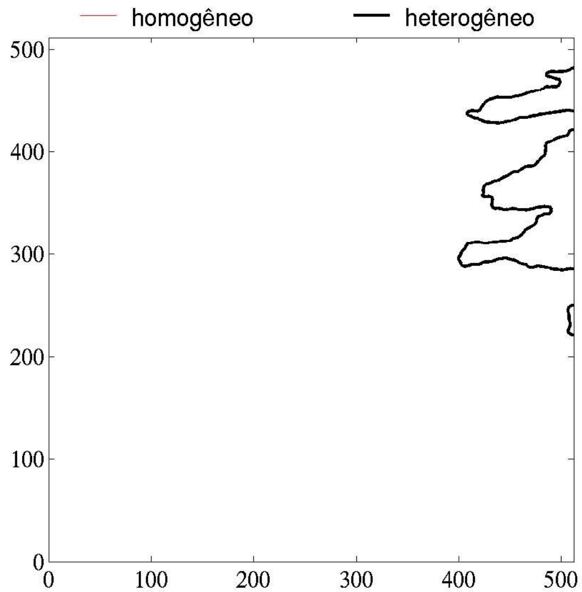

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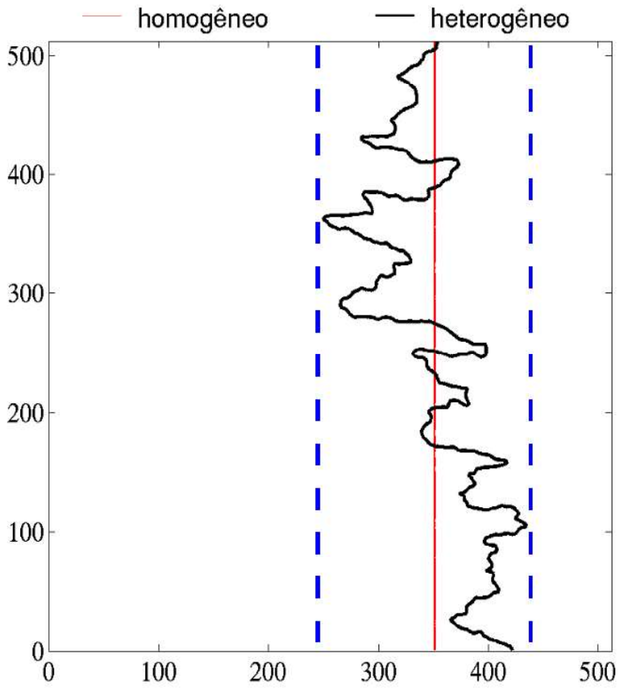


Figure 2: Macroscopic mixing length.

Mixing length

- Furtado e Pereira (2003)
  - Initial value problem:

$$\frac{\partial c}{\partial t} + \mathbf{u} \cdot \nabla c = 0, \tag{6}$$

$$c(x,0) = \begin{cases} c_- & \text{if} \quad x < 0 \\ c_+ & \text{if} \quad x \geq 0 \end{cases} \tag{7}$$

Mixing length

- Mixing length $\ell(t)$:

$$\ell(t) = \frac{1}{(c_- - c_+)} \int_0^{L(t)} |\overline{c}_e(x,t) - c_H(x,t)| dx \tag{8}$$

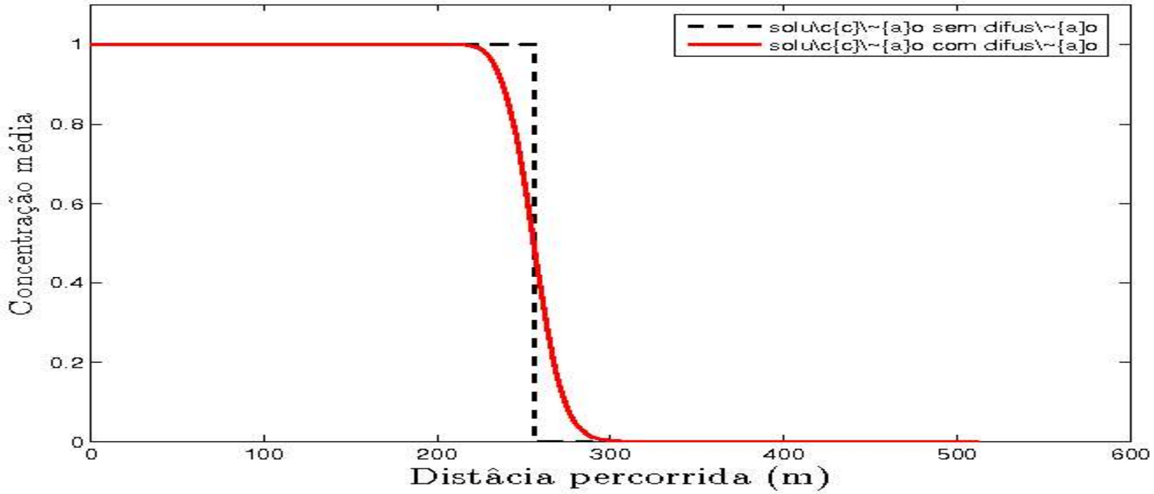


Figure 3: Concentration profiles $\mathcal{M}(t)$.

Mixing length growth

- Assimptotic mixing length growth (Glimm e Sharp, 1991):

$$\ell(t) \sim t^{\gamma} \qquad \text{onde} \qquad \gamma = \max\{1/2, 1 - \beta/2\} \tag{9}$$

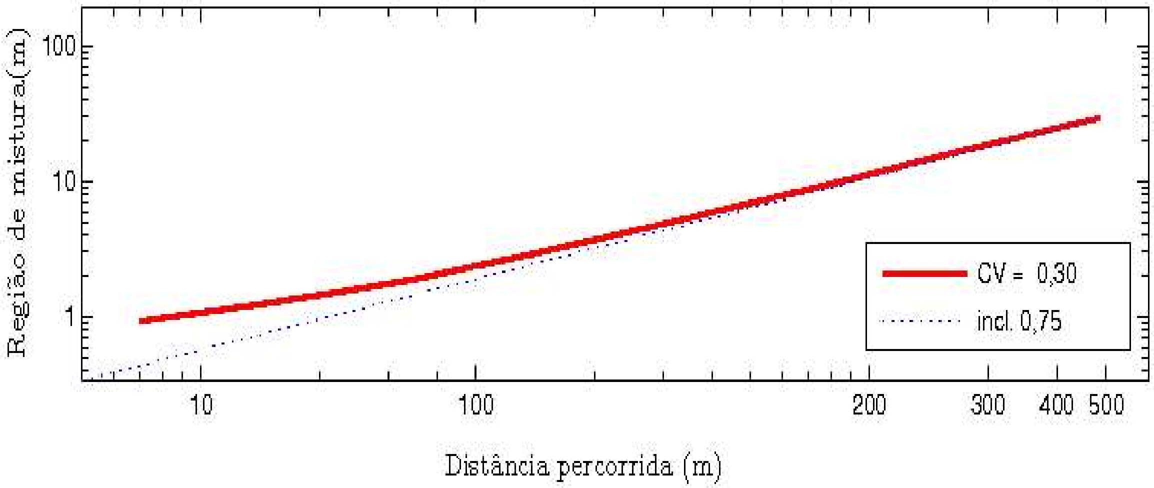


Figure 4: Mixing length $\ell(t)$ in function of the distance, in log-log scale.

Numerical approximation

- The linear problem:

System of equations

$$\begin{cases} \phi \dfrac{\partial c}{\partial t} + \mathbf{u} \cdot \nabla c = 0 & \text{FD} \\ \mathbf{u} = \dfrac{-k(\mathbf{x})}{\mu} \nabla p, \qquad \nabla \cdot \mathbf{u} = 0 \end{cases} . \tag{10}$$

Flow equation

The model problem is a second-order linear elliptic equation derived from the following system of equations:

$$\mathbf{u} = \frac{-k(\mathbf{x})}{\mu}\nabla p, \qquad \nabla.\mathbf{u} = 0, \qquad in \quad \Omega. \tag{11}$$

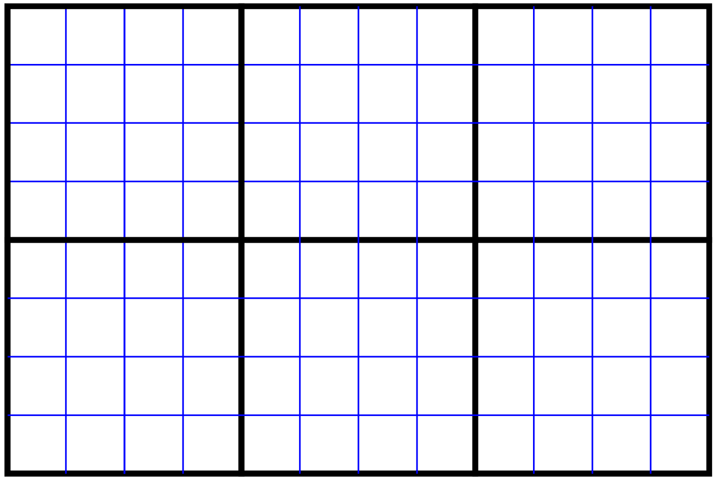
Motivation

- We use a nonoverlapping domain decomposition procedure combined with a mixed finite element discretization to develop the multiscale method *MuMM*.

A. Francisco, V. Ginting, F. Pereira and J. Rigelo, *Design and implementation of a multiscale mixed method based on a nonoverlapping domain decomposition procedure,* Mathematics and Computers in Simulations, 99 (2013) 125–138.

Domain decomposition

- For instance, the domain Ω is divided into subdomains $\{\Omega_j\}, \quad j = 1:6$.



Coarse Domain Decomposition

fine grid: 8×12
coarse grid: 2×3

Domain decomposition

- Consider a subdomain Ω_j. The local problem is given by

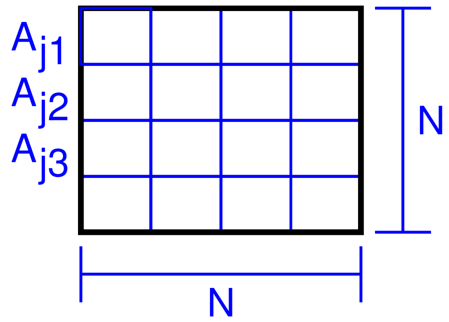
$$\nabla.(\frac{-k(\mathbf{x}_j)}{\mu}\nabla p_j) = 0 \quad in \quad \Omega_j, \tag{12}$$

$$p_j = A_{ji} \quad on \quad \Gamma_j. \tag{13}$$

The multiscale basis functions

- Given the Dirichlet boundary values A_{ji}, $i = 1, ..., 16$, the solution of the local problem in Ω_j is given by

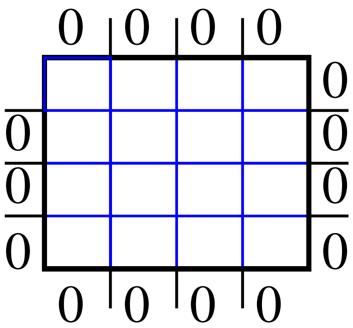
$$p_j = \sum_{i=1}^{16} A_{ji}\psi_{ji}$$



The multiscale basis functions

- The multiscale basis functions ψ_{ji}, $i = 1, ..., 16$, are defined by solving the free-divergence local problem with the canonical boundary condition. (Ganis and Yotov, 2009)

$\psi_b = 1 \rightarrow$



ψ_{j1}

The multiscale basis functions

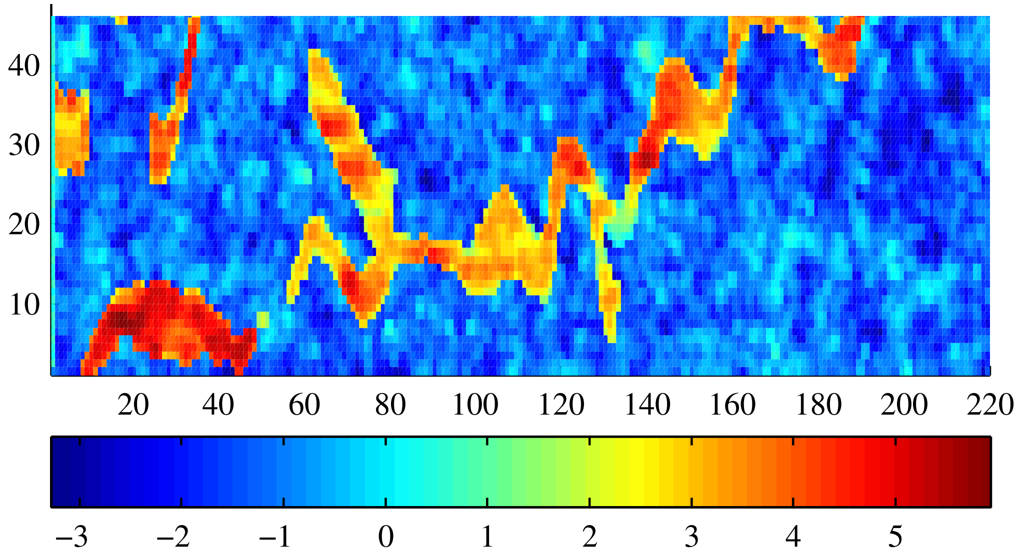
Advantage :

- Avoid the direct solution of the local problems.

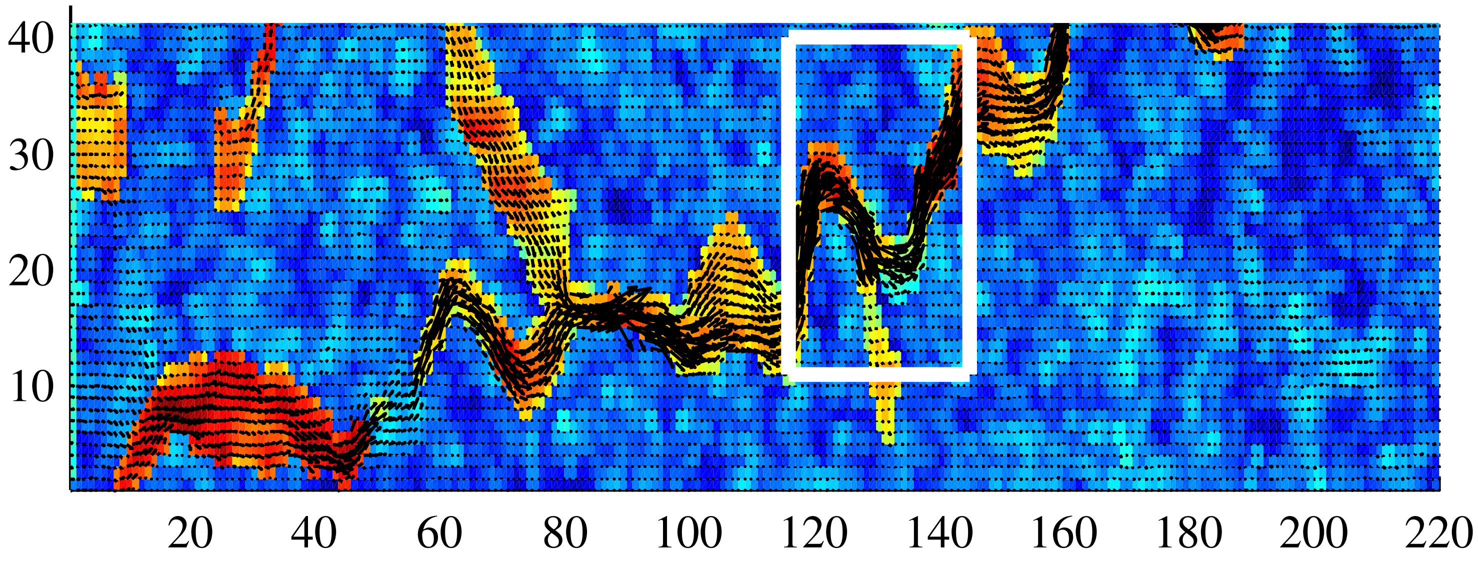
Problem :

- We have to compute 16 basis functions for each subdomain!

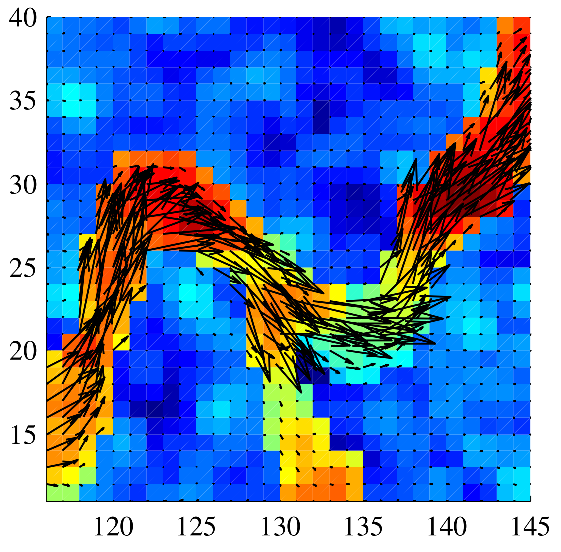
Permability field from a SPE10 model



MuMM velocity field solution



Zoomed window



Transport equation

- Forward Integral-Tube Tracking (FIT) (Aquino et al., 2011)
  - not use a solution of the Riemann problem;
  - use the idea of integral tubes introduced by Douglas et al. (2000);
  - is virtually free of numerical diffusion;
  - provide excellent computational efficiency.

Divergence form

Initial value problem

$$\nabla_{t,\mathbf{x}} \cdot \begin{pmatrix} \phi c \\ \mathbf{u} c \end{pmatrix} = 0, \qquad \mathbf{x} \in \mathbb{R}^2 \tag{14}$$

$$c(\mathbf{x}, 0) = c_0(\mathbf{x}). \tag{15}$$

Eq. trans.

Comments (Cur. int.)

Conservation identity

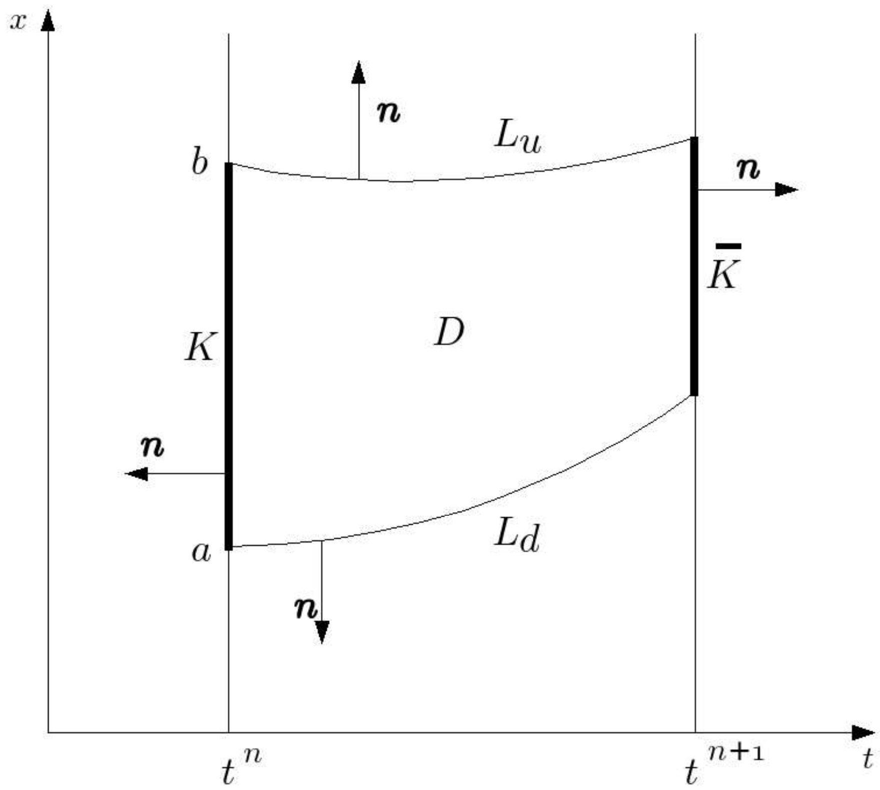
$$\int_D \nabla_{t,x} \cdot \binom{\phi c}{uc} dxdt = 0$$

$$\int_{\partial D} \binom{\phi c}{uc} \cdot \mathbf{n} \;\; dS = 0$$

$$\int_{\phi \overline{K}} c(x, t^{n+1})dx - \phi \int_K c(x, t^n)dx = 0$$

$$\int_{\overline{K}} c(x, t^{n+1})dx = \int_K c(x, t^n)dx. \tag{16}$$

- Therefore, the mass is locally conservative.



Construction of the integral tubes in space-time domain

Initial value problem (integral curve)

$$\frac{d\mathbf{y}}{dt} = \frac{\mathbf{u}}{\phi},$$

hspace15pt $t^n \leq t \leq t^{n+1}$, (17)

$$\mathbf{y}(\mathbf{x}^n, t^n) = \mathbf{x}^n. \tag{18}$$

Solving the problem above we obtain for each $\mathbf{x} \in \partial K$

$$\overline{\mathbf{x}}^{n+1}(\mathbf{x}^n) = \mathbf{y}(\mathbf{x}^n, t^{n+1}). \tag{19}$$

Comments (Cur. int.)

Numerical scheme

- In the numerical construction of $\overline{K}$ it is employed a family of points $\mathbf{x}_k^n$, $k = 1, ..., m$ that are met on the boundaries of ∂K, and the solution of Eq. (18) will be approximated by

Numerical scheme

$$\overline{\mathbf{x}}_k^{n+1} = \mathbf{x}_k^n + \frac{\mathbf{u}}{\phi}(\mathbf{x}^n)\,\Delta t, \qquad k = 1, ..., m. \tag{20}$$

Numerical scheme

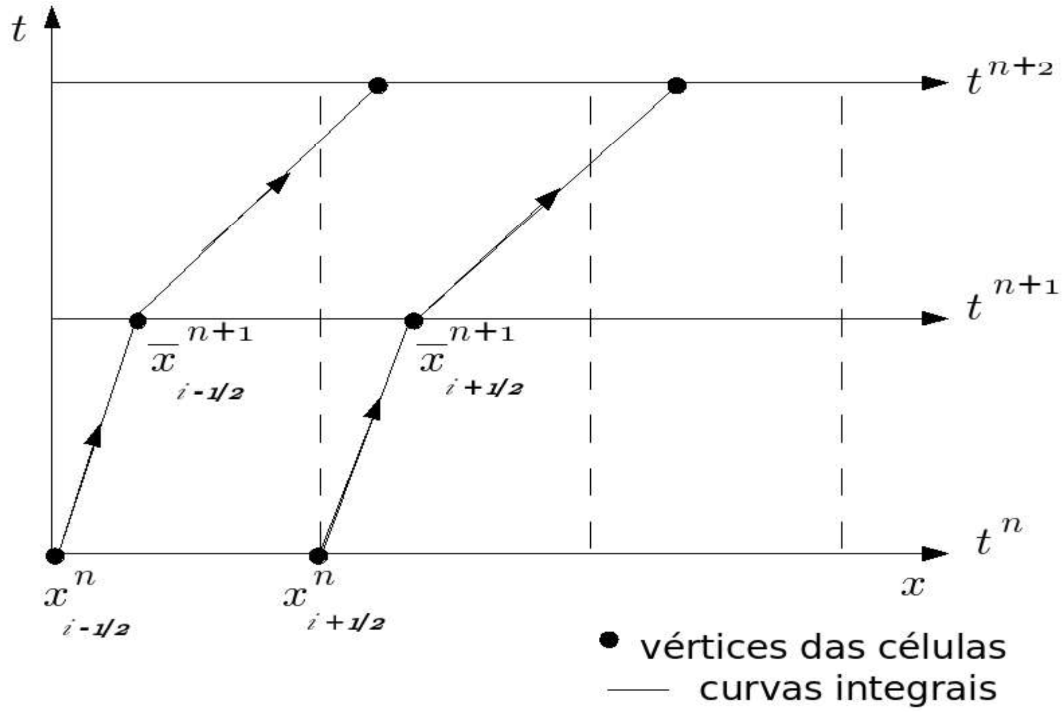


Figure 5: Integral curves for an one-dimensional space.

The integral tube in bidimensional space

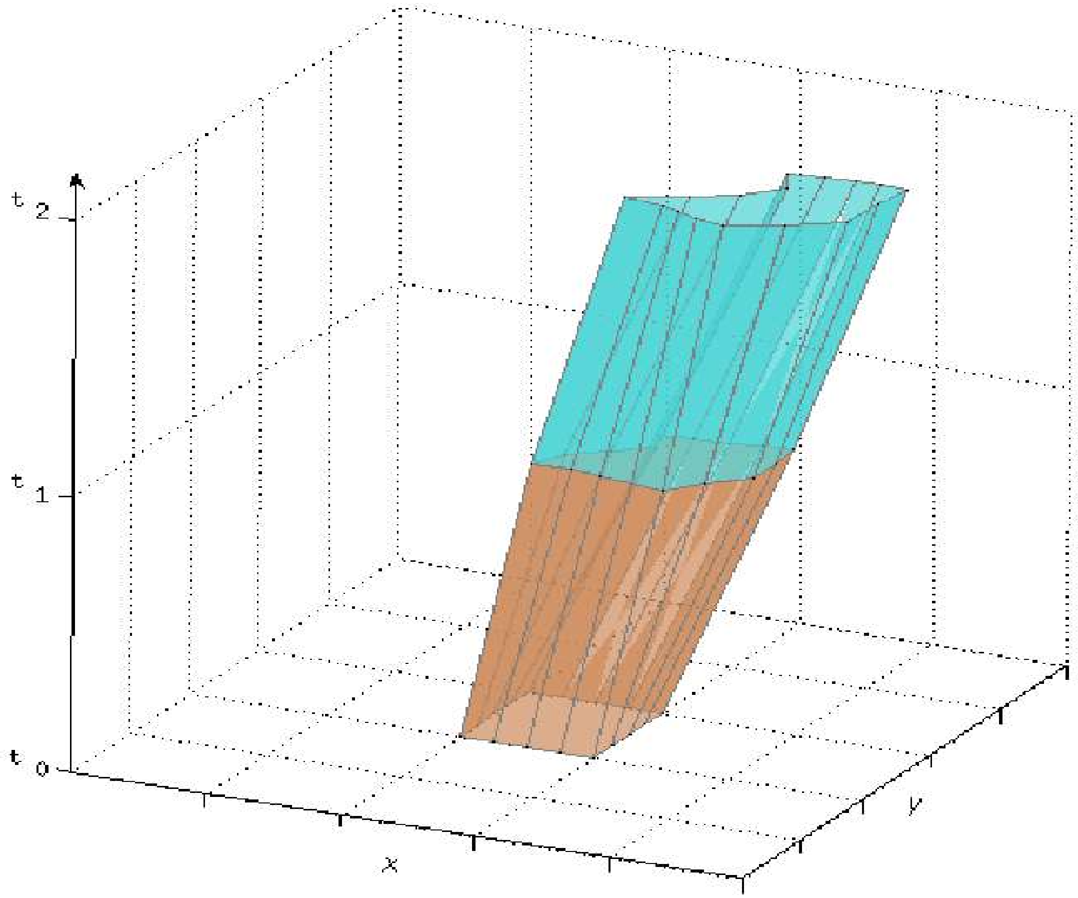


Figure 6: Integral tube D for a two-dimensional space.

Method implementation

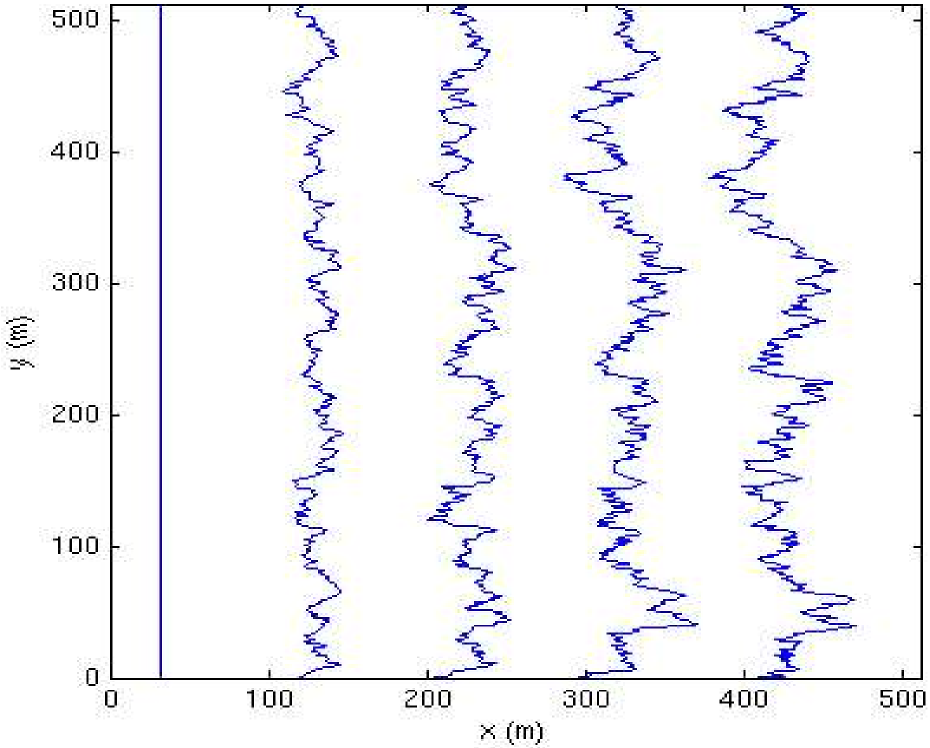


Figure 7: Position of the front of the tracer fluid at t^0, t^1, t^2, t^3 and t^1 days .

Numerical results

N-T method (Nessyahu e Tadmor, 1990)

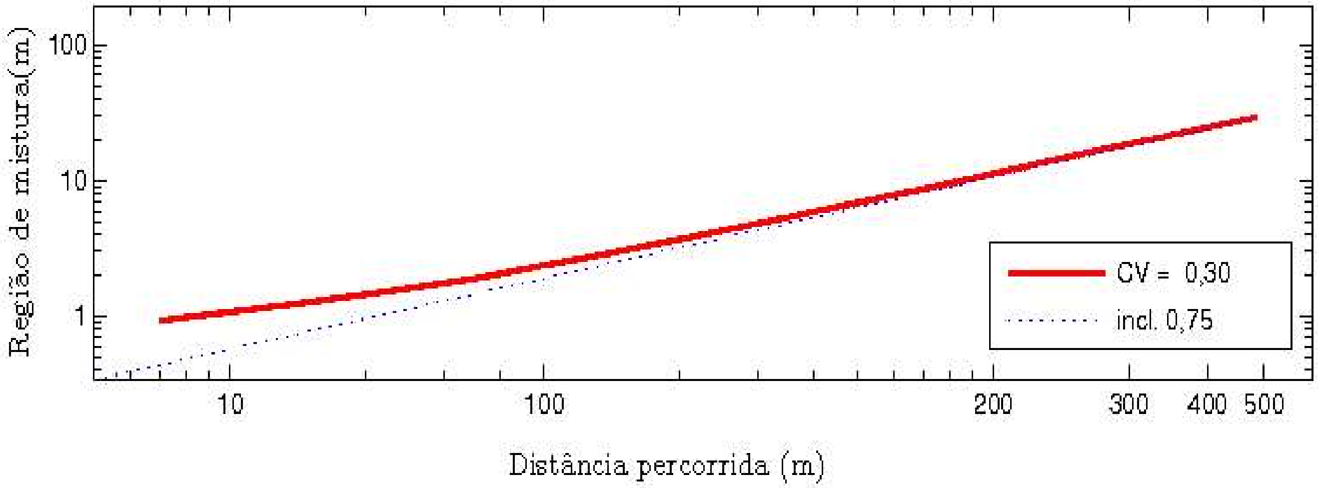


Figure 8: Mixing length $\ell(t)$ in function of the distance, in log-log scale, for a value of $\rho = 0,59$. N-T with computational grid 1024 $\times$ 1024 elements

Numerical results

FIT method (Aquino et al., 2007)

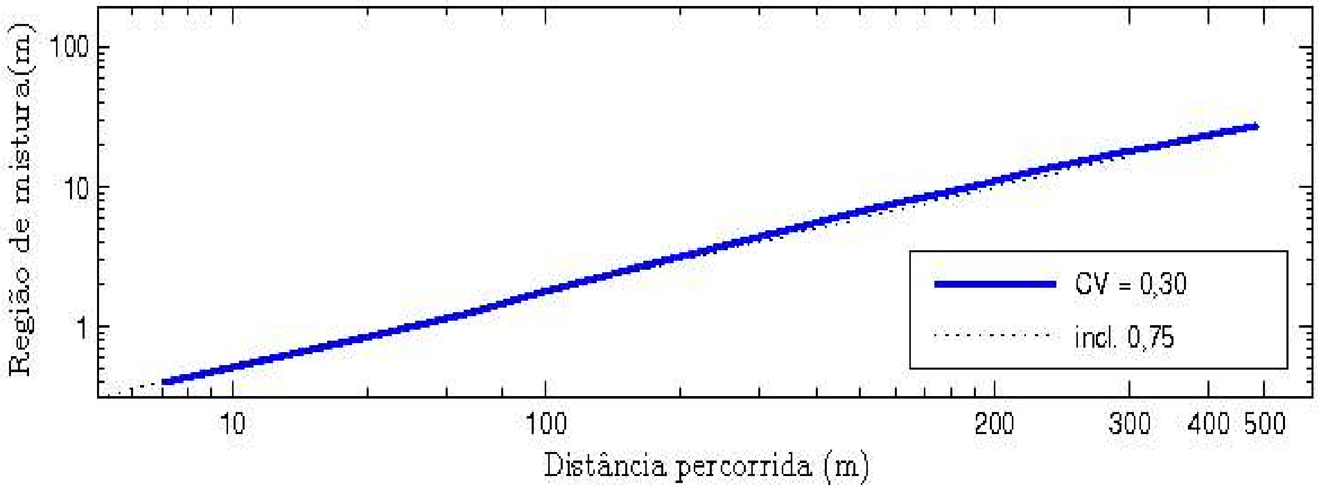
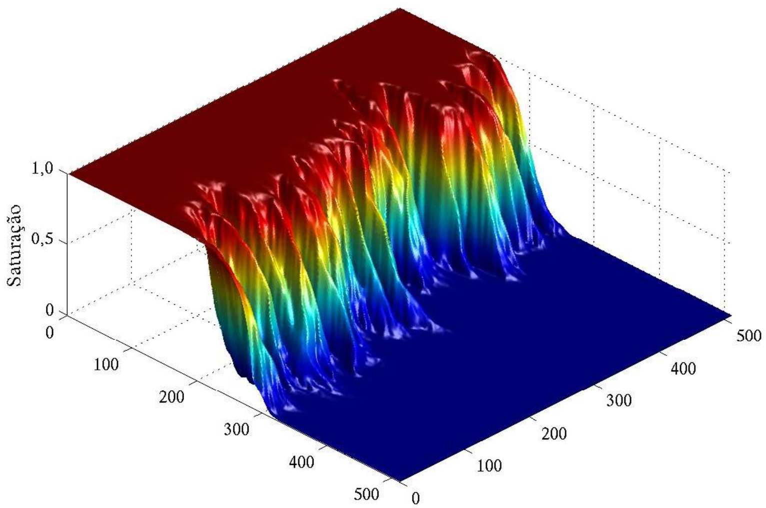


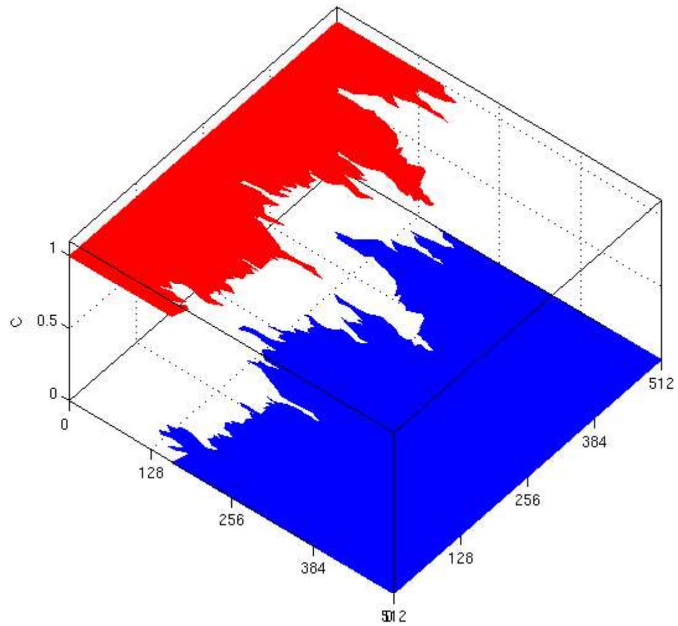
Figure 9: Mixing length $\ell(t)$ in function of the distance, in log-log scale, for a value of $\rho = 0,59$. FIT with computational grid 1024×1024 elements

Front of the tracer fluid

N-T method



FIT method



Future works

- The non-linear problem:

System of equations

$$\left\{\begin{array}{l} \phi\dfrac{\partial c}{\partial t} + \mathbf{u}(c)\cdot\nabla c + \nabla\cdot(-D\nabla c) = 0 \\ \\ \mathbf{u} = \dfrac{-k(\mathbf{x})}{\mu(c)}\nabla p, \qquad \nabla\cdot\mathbf{u} = 0 \end{array}\right. . \tag{21}$$

FD

Future works

- Proposals;
  - Operator splitting scheme;
  - Velocity extrapolation for transport microsteps; and
  - Approximate scheme for concentration gradients.

THANK YOU!

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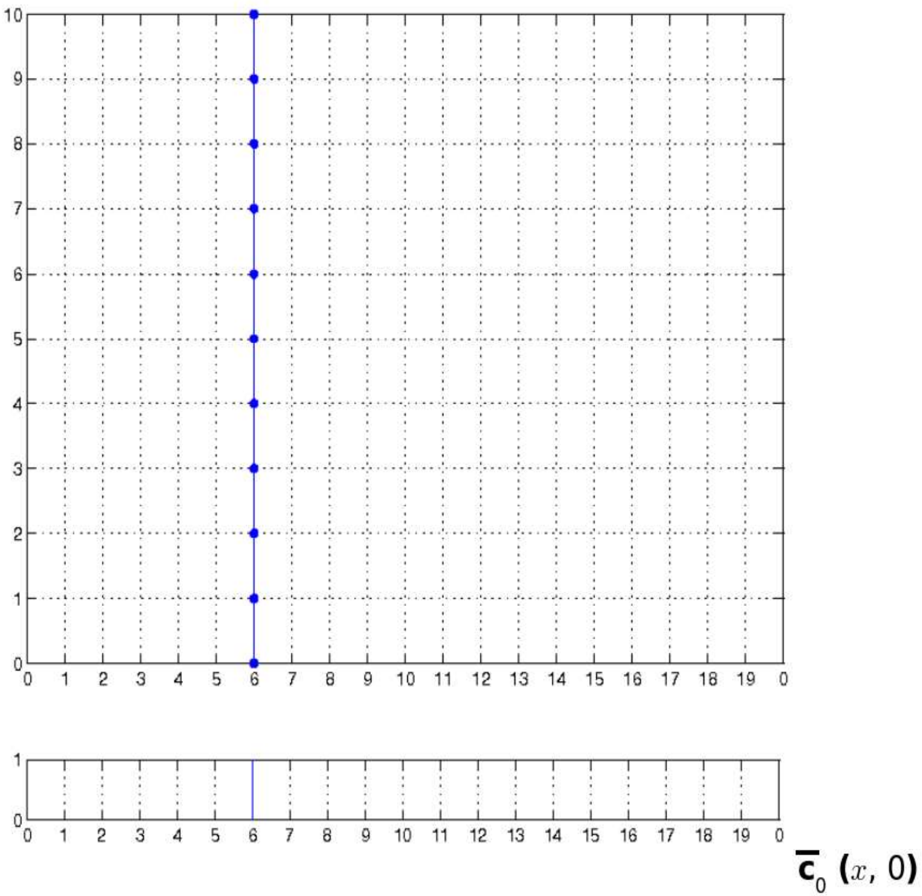
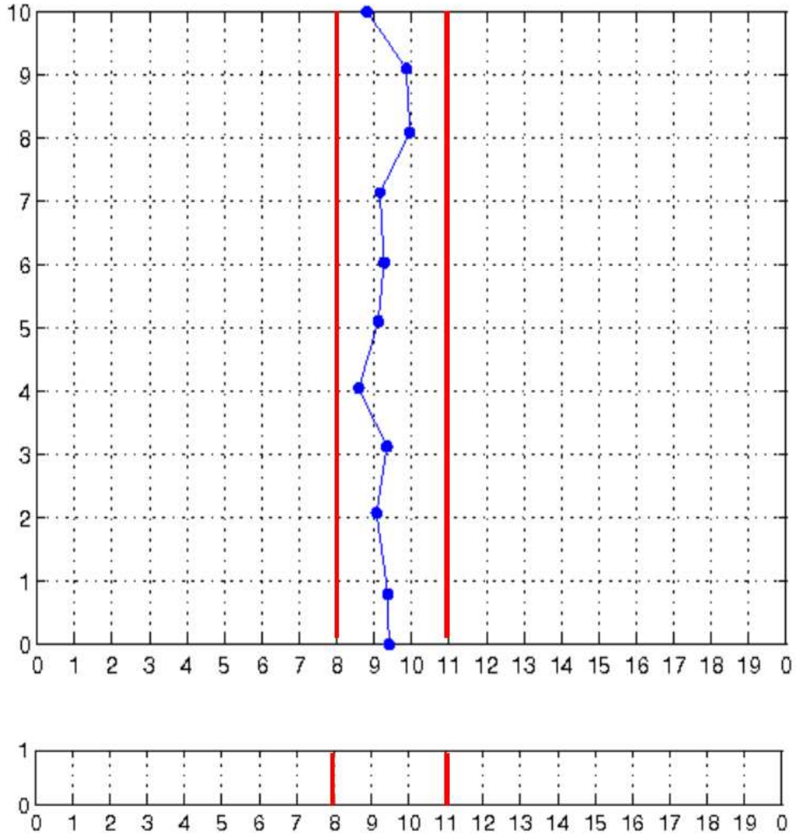
Concentration calculus



Concentration calculus



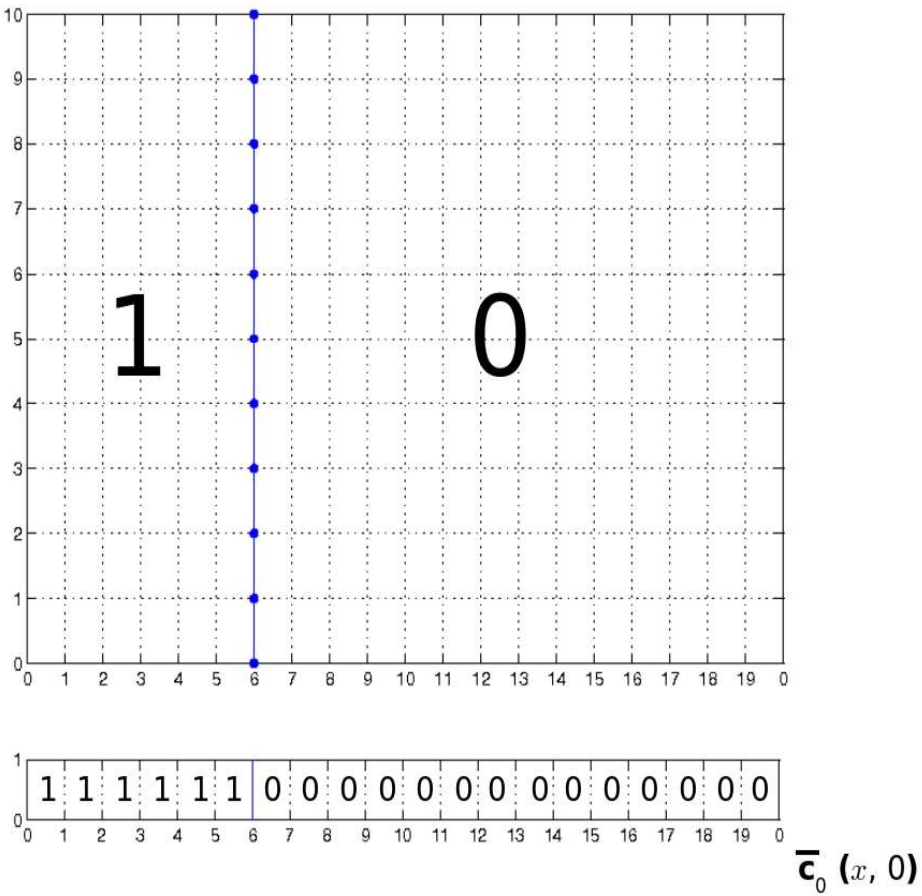
$\overline{\mathbf{c}}\,(x, t^n)$

Concentration calculus

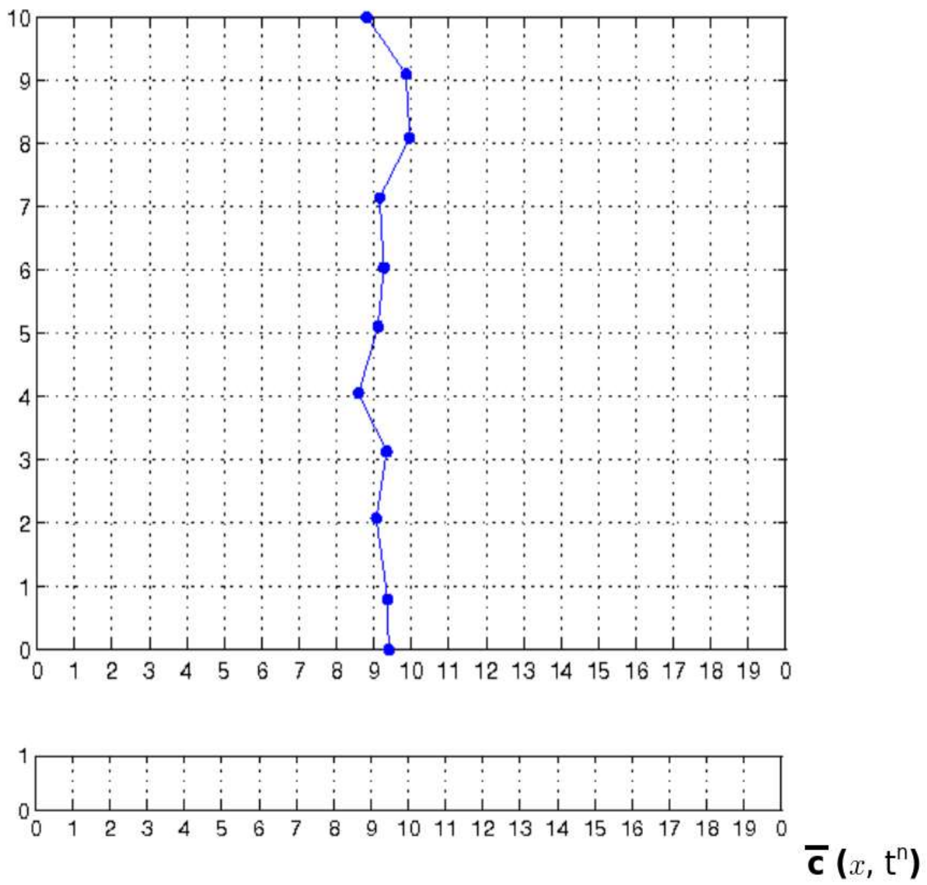
- Riemann problem

$$c(\mathbf{x},0)=\begin{cases}1, & \text{if} \quad x<3\ m\\ 0, & \text{if} \quad x\geq 3\ m\end{cases} \tag{22}$$

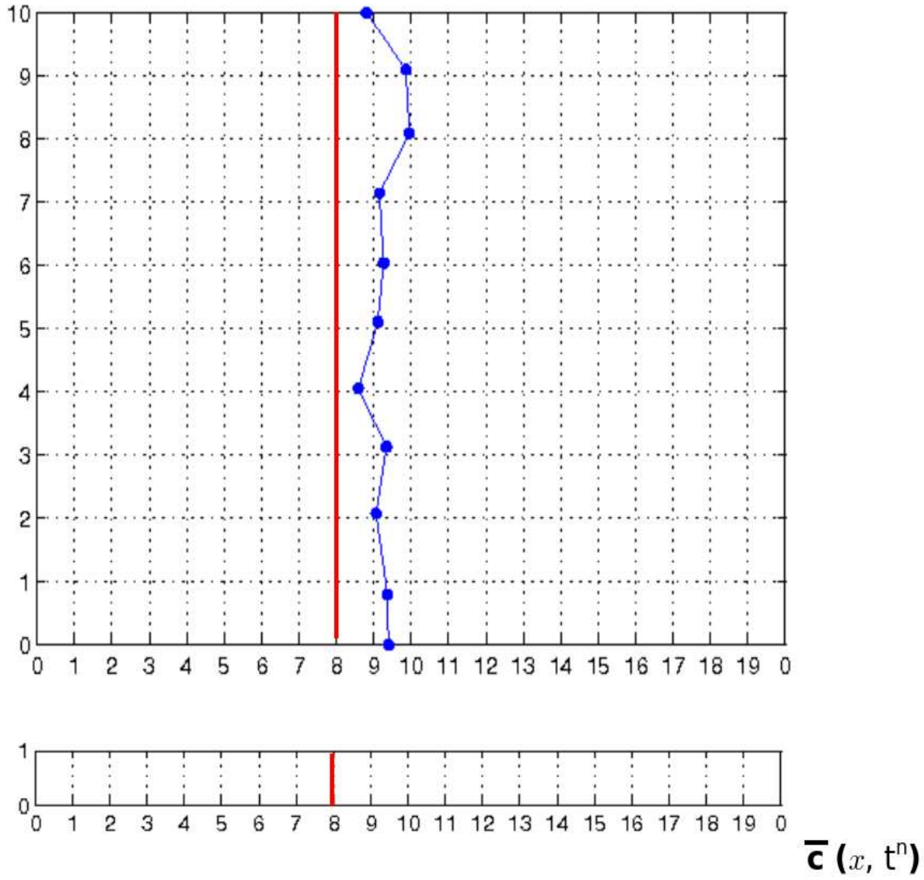
Concentration calculus



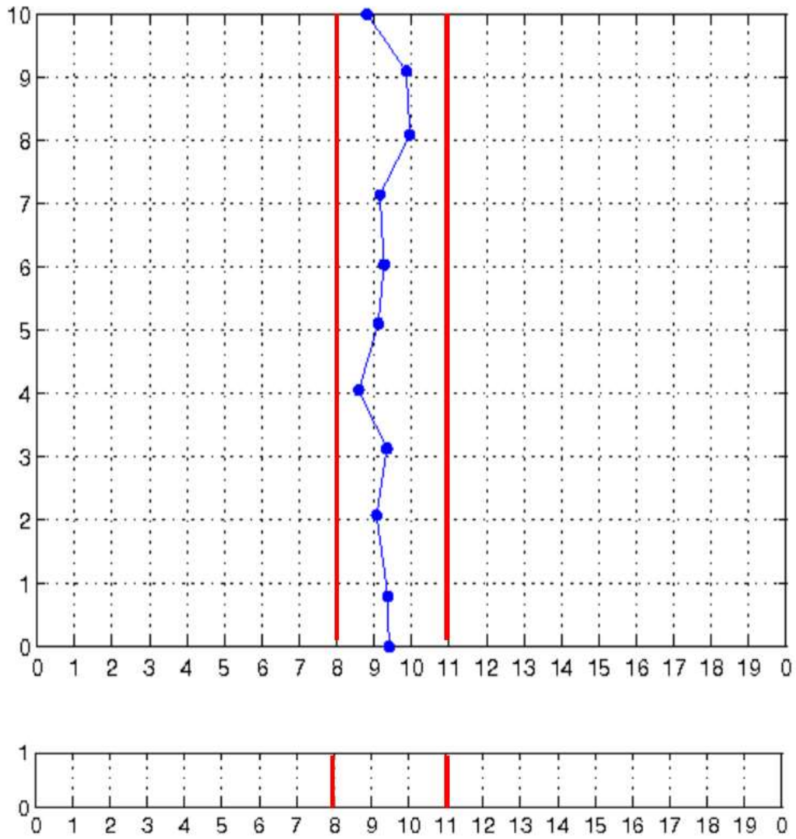
Concentration calculus



Concentration calculus

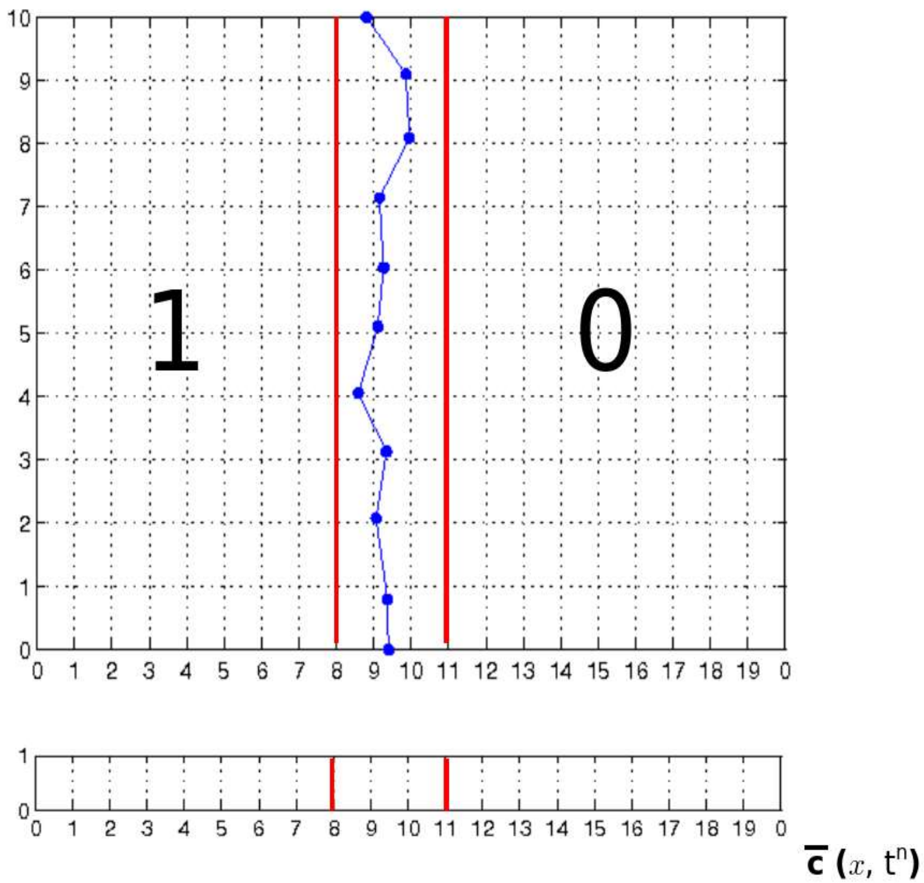


Concentration calculus

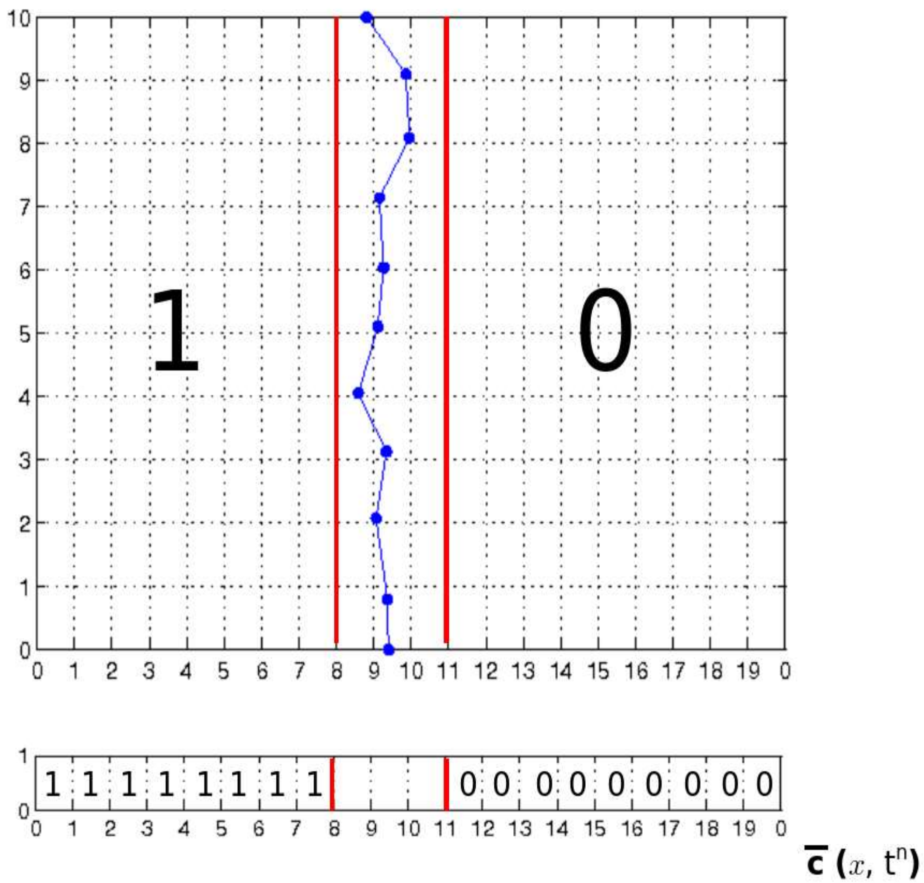


$\overline{\mathbf{c}}\ (x, t^n)$

Concentration calculus



Concentration calculus



Concentration calculus